MURNGIN: A MATHEMATICAL SOLUTION

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MURNGIN

A MATHEMATICAL SOLUTION

PIN-HSIUNG LIU

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PREFACE

In this paper I present the result of my mathematical study of the Murngin system. The work was began in spring 1967. During that summer a short essay "A note on the Murngin system" was published in the Bulletin of the Visiting Scholars Association. Harvard-Yenching Institute, Harvard University, China Branch Vol. 5-6 (1967) and appeared also in the Newsletter of Chinese Ethnology No. 7 of the same year. By the end of the following summer, the first draft of the present study in Chinese was prepared. The major part concerned with the section system was summarized in English under the title of "Formal analysis of prescriptive marriage system: the Murngin case" for presentation at the VIIIth International Congress of Anthropological and Ethnological Sciences held in Tokyo and Kyoto, September 1968. In this résumé a new mathematical device was adopted, which permitted me to revise the original draft. This second draft was published under the title of "Mathematical study of the Murngin system" in Chinese in the Bulletin of the Institute of Ethnology, Academia Sinica, No. 27 in 1969. The present English version makes some changes and refinements necessary.

In this paper I deal with mathematical models, but since I am a social anthropologist, I am fully aware that the method by which the results are reached may not be as concise and direct as that of a mathematician.

The mathematical analysis of the Murngin System proves that the joint operation of anthropology and mathematics leads to the solution of a hitherto insolvable problem. The application of mathematical methods is in fact not an unsurmountable barrier facing the anthropologist as generally assumed. It is hoped that mathematicians may become aware of the dilemma and lend their vital assistance to anthropologists, for it is only by a combination of the two disciplines that solutions to such problems may be reached. For a general discussion of this question, attention is drawn to my paper "Theory of groups of permutations, matrices and kinship: a critique of mathematical approaches to prescriptive marriage systems" which was published in the Bulletin of the Institute of Ethnology, Academia Sinica, No. 26 in 1968, is included here as an appendix to the present study.

During the entire period of research, the work was supported by the Harvard-Yenching Institute, Harvard University. The author is much obliged to his colleagues of the Institute of Ethnology, Academia Sinica, for their helpfulness throughout this study. Special thanks are due to Mrs. Inez de Beauclair and Mr. Raleigh Ferrell who kindly examined the English text. Publication expenses were generously provided by the National Council for the Development of Sciences.

PIN-HSIUNG LIU

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WORKING HYPOTHESIS AND METHOD

One of the most remarkable achievements in recent kinship study is the establishment of kinship algebra, a long-pending problem for social anthropology. As early as the end of the last century, social anthropologists recognized the applicability of mathematics to the analysis of the Australian section systems regulated by prescribed marriage rules. It was Francis Galton (1889) and Émile Durkheim (1898) who first proposed the theory of 'double descent' as a clue to the mathematical study of kinship. Since that time not a few scholars have been engaged in this study, but unfortunately, owing to their restricted mathematical knowledge, kinship algebra was never realized as a established science.

It was André Weil who first applied pure algebra to the study of certain types of marriage laws, namely, the section system. Weil proposed the following three rules as basic properties of the matrilateral cross-cousin marriage system (Lévi-Strauss 1969: 221-222):

- (A) For any individual, man or woman, there is one and only one type of marriage which he (or she) has the right to contract.
- (B) For any individual, the type of marriage which he (or she) may contract depends solely on sex and the type of marriage from which he (or she) is descended.
- (C) Any man must be able to marry his mother's brother's daughter.

Based on the special character of the 'marriage types' proposed by him, in which indication of the marriage type of the children's generation is nothing but a rearrangement of that of the parent's generation, Weil points out that the theory of groups of permutations is applicable to the study of the section system. Thus this method is applied to prove that the four-section system proposed by Claude Lévi-Strauss as the implicit system of the Murngin could meet rule (C). For the Murngin's present eight-subsection system, owing to the contradiction between its marriage regulations and rule (A), Weil introduces another mathematical device, the addition of an *n*-tuple modulo two system, to demonstrate that Lévi-Strauss's hypothesis is mathematically constructable. Weil's unique suggestion has shaped the current mode of mathematical approaches to kinship study.

Robert R. Bush, extending Weil's method, concludes that the algebra of permutations, special topics in group theory, matrix algebra, and operator algebra are appropriate for the study of the section system. Thus Bush introduces the concept of a mathematical 'operator' demonstrating that 'permutation matrices' are an effective tool for kinship analysis. One of the extraordinary merits of this method is the production of identity operators and other equations, which means the formulation of generation cycles of descent lines or marriage rules for the given society in mathematical formulae. (See White 1963, Appendix 2.)

After Bush, Kemeny, Snell and Thompson contributed to an algebraic analysis of the societies to be investigated an integrated set of axioms as follows (Kemeny, Snell and Thompson 1956: 343):

- Axiom 1. Each member of the society is assigned a marriage type.
- Axiom 2. Two individuals are permitted to marry only if they are of the same marriage type.
- Axiom 3. The type of an individual is determined by the individual's sex and by the type of his parents.
- Axiom 4. Two boys (or two girls) whose parents are of different types will themselves be of different types.
- Axiom 5. The rule as to whether a man is allowed to marry a female relative of a given kind depends only on the kind of relationship.
- Axiom 6. In particular, no man is allowed to marry his sister.
- Axiom 7. For any two individuals it is permissible for some of their descendants to intermarry.

Both method and axioms are revised by Harrison C. White (1963). Perceiving that marriage type is not a concept to be found in either the field notes of anthropologists or the thinking of members of the societies, White adopts two new operators or generators. He represents the transformation of husband's section into wife's section by one matrix, and the transformation of father's section into children's section by another, instead of having one matrix representing the transformation of parent's marriage type into son's type, and another similar matrix to represent daughter's marriage type. Meanwhile, Kemeny-Snell-Thompson's axioms are revised as follows (1963: 34-35):

- The entire population of the society is divided into mutually exclusive groups, which we call clans. The identification of a person with a clan is permanent. Hereafter n denotes the number of clans.
- 2. There is a permanent rule fixing the single clan among whose women the men of a given clan must find their wives.
- 3. By rule 2, men from two different clans cannot marry women of the same clan.

- 4. All children of a couple are assigned to a single clan, uniquely determined by the clans of their mother and father.
- 5. Children whose fathers are in different clans must themselves be in different clans.
 - 6. A man can never marry a woman of his own clan.
 - 7. Every person in the society has some relative by marriage and descent in each other clan: i.e., the society is not split into groups not related to each other.
- 8. Whether two people who are related by marriage and descent links are in the same clan depends only on the kind of relationship, not on the clan either one belongs to.

In White's axioms the term *clan* is used instead of *section*, but this substitution might cause conceptual confusion, and its unfitness has already been pointed out by Russell M. Reid (1967: 171). He insists that his proposed 'marriage cycles' is the essential feature of the model resulting from White's eight axioms, and therefore he proposes a ninth axiom as follows:

All marriage cycles in the same system must contain the same number of segments.

Though the axioms and methods are incessantly being refined and improved, they still contain some flaws in themselves, so the effective range of applicability is still limited. First, as recognized by the mathematician himself, the method is not applicable to the analysis of societies practicing matrilateral cross-cousin marriage, such as the Murngin or Prums, owing to the contradictions between their marriage rules and the axioms mentioned above (White 1963: 145). Secondly, such systems as those of uncle/niece marriage, or absurdities such as father/daughter or mother/son marriage and others are taken for matrilateral cross-cousin marriage systems in mathematicians' treatments (Liu 1968). Once the deficiencies of the method are

clarified, methodological improvement should follow at once. However, no matter how far the method may be refined, the Murngin problem remains insolvable as long as the axiomatic contradiction persists. Without the establishment of new axioms or hypotheses and a new proposal of mathematical method, the Murngin problem will remain an enigma forever.

In the author's recent study of kinship, the mathematical model of kinship or genealogical space is discussed (Harvey and Liu 1967). Genealogical space is the structural frame upon which all kinship systems depend to exist, but up to the present its basic properties have been regarded as self-evident and it has been rarely discussed (e.g., Fisher 1960). 'Kinship category' is a new concept concerned with one part of the genealogical space, wherein all kin relationships are reduced to the two basic units 'parent' and 'child', and are expressed by the products of the two units as generators. The kinship categories represented by the numerical notation system are computable, and in fact the 'generation transition' of the kin relationship itself is a kind of typical binary operation. Thus we can point out that a set of kin groups composed of the kinship categories possesses the following properties of algebraic group theory:

- (1) Identity: 00 is the identity unit or unit element.
- (2) Inverses: Each element has its inverse in the set.
- (3) Associativity: This property is satisfied by the binary operation.
- (4) Group equations $a \cdot x = b$ and $x \cdot a = b$ are solvable. But this group is not commutative and its elements are productive without any limitation, so it may be called an unfinite non-Abelian group.

However, kinship category is not the only kinship structure derivable from genealogical space, there are also many other different forms. For example, 'kinship type' is a unit well-known to anthropologists, and is customarily expressed by the combination of language-oriented notation system such as F, M, B, Z, S, D, etc. If the operators or generators producing kinship systems were to be explored exhaustively, then we could compose all sorts of structures within the genealogical space for the analyses of any complicated structural problems.

The kinship structures of the Australian aborigines are well known for their unique and complicated genealogical space characterized by socalled 'double descent' or 'section system'. Owing to the basic structural differences from other societies none of the previously employed devices are effective. To establish the spacial forms of genealogical space inherent in prescriptive marriage systems, to explore its mathematical properties and to compose the mathematical models of kinship structures are the main aims of this paper. For this purpose I propose here two basic units: father-child link and mother-child link. The former is represented by m and the latter by f, which are used as generators for the analysis of the section systems.

No society in Australia is more controversial than that of the Murngin, which has been studied and discussed by social anthropologists for almost forty years and has been considered nearly an insolvable problem (Barnes 1967). There may be many reasons for this failure. But the main cause is not due to the lack of data or the missing of some crucial facts as Barnes thought. In fact no society has ever been so intensively investigated by trained social anthropologists accumulating a bewildering wealth

of materials as the Murngin. What then is the real cause of the failure? It can be no other than the insufficiency of the kinship theory applied. Seeing the conceptual confusion caused by Lévi-Strauss and Leach in treating uncle/niece marriage as a form of matrilateral cross-cousin marriage (Liu 1968), we can not help but admit that it is not the aborigines but the anthropologists who are to be blamed for the theoretical impasse.

In the following discussion, some characteristics of prescriptive marriage systems are shown by structural models of several typical and conjectual systems depicted in Figure 1. Here the mathematical Cayley diagram is adopted. The solid line represents generator m and the dotted line represents generator f. The arrow-head indicates the transition of generation from ascending to descending generation. If the generation transition is reversible, no sign is attached to the line.

Figure 1a, b and c represent three well-known societies, Arunta, Ambrym and Kariera, characterized by bilateral crosscousin marriage systems. The Arunta practice second crosscousin marriage, Kariera first cross-cousin marriage, and the Ambrym are in the middle, with first cross-cousin once removed, that is, an oblique marriage. Here the Korean 'inch system' is adopted to show kinship distance. Applying the Harvey-Liu system, kinship distance Kd is expressible in following formula: Kd=Sxy. The kinship distance between spouses decreasses 'one inch' from society to adjacent society, say, from Arunta to Ambrym and Ambrym to Kariera. If the decrease of kinship distance continues with the same spacing, following first cross-cousin marriage we could get 'uncle/niece marriage' and 'sibling marriage' in sequence. Supposing sibling marriage

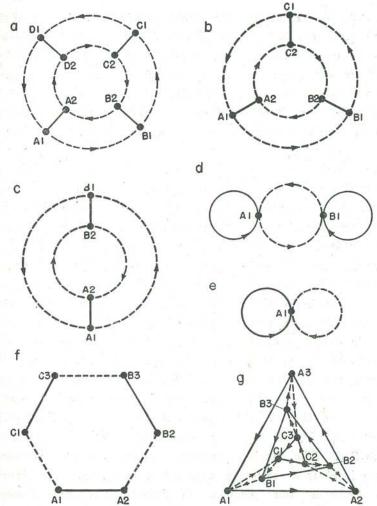


FIG. 1. Structural models of prescriptive marriage systems.

a. Arunta b. Ambrym c. Kariera d. Uncle/niece marriage e. Sibling marriage f. Patrilateral cross-cousin marriage g. Matrilateral cross-cousin marriage

as the starting point or zero point and defining the different degrees of kinship distance with other societies as their 'marriage degree Md', which is expressible in the following formula: Md = Sxy-2, we could then list the components such as patri-line, matri-line, section and marriage degree of these five types of societies respectively as follows:

TABLE 1. STRUCTURAL COMPONENTS IN SECTION SYSTEM

a	Arunta	Ambrym	Kariera	Uncle/ niece* marriage	Sibling marriage	
Section	8	6	4	2	1	
Patri-line	4	3	2	2	1	
Matri-line	2	2	2	1	1	
Marriage degree	4	3	2	1	0	

^{*} Aunt/nephew marriage is also constructable with 2 sections, 2 matrilines, 1 patri-line and 1 marriage degree.

Objection may be raised to our listing in Table 1 of uncle/niece and sibling marriages as on a par with bilateral cross-cousin marriage. But from the structural point of view, it is not hard to find that all the above-listed societies are constructed from the same context as shown in Figure 1 and Table 1. The number of descent lines of sibling marriage is indicated as 1 for convenience, but in fact it constitutes a non-divisional stage, so it might be better to use null to represent it. Moreover, Table 1 indicates other rules: first, the number of sections and marriage degrees have a consistent relationship, that is, the number of the former is always two-fold that of the latter. Second, the patriline and matri-line intersect each other, so the number of each

descent line constitutes the number of the opposite descent-line's generation cycle⁽¹⁾.

Theoretically, a unilateral cross-cousin marriage system is never established without at least three descent lines or groups to form 'connubium circulation' or 'marriage alliances'. Figure 1f shows a structural model of a patrilateral cross-cousin marriage system consisting of three patri- and three matri- descent lines. Each of the descent lines is regulated by a 2-generation cycle as its distinctive feature, independently of the increase of the number of descent lines participating.

Figure 1g represents a matrilateral cross-cousin marriage system composed of three patri-lines, and simultaneously of matri-lines in equal number, both regulated by 3-generation cycles. In principle the number of marriage cycles coincides with the number of descent lines, but this law is not invariable (see Chapter 4 and 5 for detailed discussion). In a unilateral cross-cousin marriage socity organized around n number of descent groups (either patri- or matri-), the number of sections producible is different in the case of patrilateral or matrilateral cross-cousin marriages; the former is 2n and the latter is n^2 .

The aim of the above cursory analysis of prescriptive marriage systems is to propose a new conception showing that the structure of three different types of cross-cousin marriage (bilateral, patrilateral and matrilateral) is regulated by different principles. As to the matrilateral system which we are going to discuse,

⁽¹⁾ Based on the above-mentioned rules, next to Arunta the following system may be conjectured: A ten-section system characterized by second cross-cousin once removed marriage, composed of 5 patri-lines and 2 matri-lines, with 5 marriage degrees (Liu 1965).

the above-mentioned principles are not the only determinant of the number of sections; other factors are also at work. For this reason, it is dangerous to judge the structure of matrilateral cross-cousin marriage systems simply by the number of the sections they contain. This is particularly true for the Murngin system which is the extreme case among them. The mathematicians' method based on the transformation of sections is most effective for the analysis of bilateral cross-cousin marriage systems, but for the unilateral cross-cousin marriage systems its effectiveness is doubtful. Structural elements additional to the sections also should be considered.

Based upon the above discussion, the following three principles—though still crude and needing refinement—are proposed as working hypotheses for the theoretical analysis of section systems:

- (1) A matrilateral cross-cousin marriage system is characterized by an *n*-generation cycle or circulation, where the marriage alliance (or circulating connubium) is derived from *n* number of hordes or exogamous units, the minimum number for *n* being 3.
- (2) Under the same condition a patrilateral cross-cousin marriage system is characterized by a 2-generation cycle, or the so-called cycle of 'alternating generations'.
- (3) A bilateral cross-cousin marriage system is characterized by the above two principles, the 'n-generation' and '2-generation' cycles, but in this case n may be 2 or more.

MURNGIN CONTROVERSY

'Murngin' is a general name given by the American anthropologist W. Lloyd Warner (1930, 1937) to the inhabitants of northeastern Arnhem Land, north Australia. As with most aborigines of that continent, the tribal consciousness of the inhabitants of this area is vague and they have no tribal name. According to Warner (1937: 17), the aborigines of this district are divided into eight branches, among which the Murngin are the largest one, so he adopted this name to designate the ethnic group.

For this many specialists disagree. T. Theodor Webb (1933: 410) denies that 'Murngin' is a tribal name, as well as the existence of eight branches; on the contrary he points out the existence of twelve totemic groups, Murngin being one of them. R. M. Berndt (1955: 84) insists that this tribe should be called 'Wulamba'. Evidently, Warner's Murngin is a misnomer. But the term 'Murngin' is now so widely accepted that Warner's usage is followed in this paper.

The population of this area is nearly three thousand, divided into sixty-odd hordes⁽¹⁾, with an average of forty to fifty indi-

⁽¹⁾ This local group is called 'clan' by Warner, who restricts the term 'horde' to an occasional economic assemblage, e.g.: "the Murngin horde is an economic group of people temporarily occupying a certain area of land. The membership of this social unit consists of from a small number of individuals to possible hundreds. It includes the members of several clans and the two moieties". (1937: 593)

viduals in a band—an exogamous unit comprised of patrilineal descent groups—roaming in an average 360 square mile territory leading a hunting and gathering life. In contrast to the simplicity of their way of life, the kinship structure constructed by the Murngin is so amazingly complicated that it has been honored as one of the greatest feats of social engineering human society has ever produced. Structural analysis of the intricate Murngin system has become the central subject of kinship theory studies ever since Warner's enormous monograph was published.

The kinship systems constructed by the aborigines scattered throughout the vast Australian continent are highly variegated. but are identified by specialists as being subordinate to or different varieties of a single general type (Radcliffe-Brown 1930: 34). Most of the aborigines divide their tribe into two, four or eight so-called 'marriage classes'. When the division is into two the classes are customary called 'moieties'; if four or eight they are called 'sections', but sometimes in the last case they are especially called 'sub-sections'. In most of the societies with section systems, whether there are four or eight sections, the marriage system is characterized by cross-cousin marriage based on direct sisterexchanges. Another prominent characteristic of this system is the so-called 'principle of alternating generations' or '2-generation cycle' revealed in the descent lines, which has been taken by specialists as the key to the secret of Australian section systems (e.g., Radcliffe-Brown 1930-1; Lawrence 1937; Lévi-Strauss 1949; Dumont 1967).

The Murngin system, possessing close similarities to that of the Arunta, is divided into two moieties and each moiety is further subdivided into four sections, making a total of eight subsections. But the mechanism controlling Murngin marriage is completely different from that of the Arunta; that is, it is not by bilateral cross-cousin marriage but by matrilateral ones. Another distinguishing feature of the Murngin system is the disappearance of the principle of alternating generations, for which is substituted a 4-generation cycle or circulation. This phenomenon puts the specialists to great embarrassment, and has never been properly explained. It is worthwhile to notice that most of the anthropologists have considered it absurd (e.g., Elkin 1933; Lévi-Strauss 1949).

In the study of matrilateral cross-cousin marriage systems, we should furthermore not ignore the system of the so-called 'circulating connubium' or 'marriage alliance', which was established by Dutch scholars at the end of the nineteenth century in the study of the Indonesian area where matrilateral cross-cousin marriage is extremely common (see Needham 1957; 1962). Murngin is one of the most distinctive Australian societies, having a matrilateral cross-cousin marriage system, so for our structural analysis the study of the circulating connubium or marriage alliance is of exceptional significance.

Some neglected problems of the Murngin controversy are discussed below based on the following questions:

- (1) What is the minimal number of hordes necessary for the formation of the Murngin's marriage alliances?
- (2) By what process is the Murngin system produced? Is there any relationship between the respective eight subsection system of the Murngin and Arunta?
- (3) Is the principle of the 4-generation cycle reasonable? If so, then by what mechanism it is regulated? Moreover, is it

really theoretically incompatible with the principle of alternating generations?

Concerning the first question, Warner has never discussed this problem directly in his work; and only in diagram did he propose to show the scope of application of kinship terms to suggest the possible existence of a marriage alliance composed of more than seven hordes or patri-lines, but ultimately how many are really necessary he never expressed clearly. W.E. Lawrence and G.P. Murdock (1949), basing their conclusions on Warner's diagram, proposed that eight patri-lines should complete the marriage alliance. This unique theory was refuted at once by A.R. Radcliffe-Brown (1951: 37) and A.P. Elkin (1953: 412), who drew attention to the fact that the applicable range of Warner's kinship terms should not be limited to seven or eight patri-lines but are extendable to limitless numbers of patri-lines. The extension of the kinship terminological network and the closing of the marriage alliance are problems apparently belonged to different dimensions, but the mixture of these two adds excessive confusion to the controversy.

Recently, White (1963: 123) has attempted to reconstruct the Murngin's marriage alliances based on Warner's report. But the description concerned with intermarriage between hordes, as stated by the author, is ambiguous and very questionable. According to the data contributed by informants, the existence of intermarriages between two hordes are widely recognized, though they have a tendency to show one side of them as more predominant or preferential (Warner 1937: 28). The marriage alliances reconstructed from these records are of course incomplete, and inevitably tend to bear the colour of bilateral cross-cousin

marriage. It is no wonder this makes White inclined to believe that the Murngin system is not that of matrilateral cross-cousin marriage as reported, but a bilateral one. But Warner in his report only states that the Murngin's marriage system is matrilateral cross-cousin marriage and never insists or suggests in any way that the system is or has the inclination to be one of bilateral cross-cousin marriage. Moreover, anthropologists who have done field work in this district are not restricted to Warner himself, but none of them has ever doubted the Murngin system to be one of bilateral cross-cousin marriage⁽¹⁾. The actual composition of the Murngin marriage alliances is still ambiguous and clarification of the real structure remains for the future.

Concerning the second question, Warner also did not discuss this problem directly but only proposed a hypothesis of 'two subsections composing one section' to reduce the Murngin's eight subsections into four sections, depicting its structure by the diagram traditionally used by Australian specialists to illustrate the Kariera system.

Among the section systems regulated by sister-exchange marriage, the Kariera's four section system is considered the most fundamental type. When first cross-cousin marriage is prohibited and second cross-cousin marriage is adopted, the original sections would divide into two for the discrimination of second cousins from first cousins. Thus each of the four sections splits into two to produce eight subsections. The Arunta are considered the typical case.

According to J.B. Barnes (1967: 2), in addition to Warner the following scholars are listed as investigators of North Arnhem Land and its neighbours: B. Spencer, T.T. Webb, D.F. Thomson, A.C. Capell, A.P. Elkin, R.M. Berndt and C.H. Berndt, F.G.G. Rose, P.M. Worsley, and L.R. Hiatt.

Judging from the outlook of traditional discipline, Elkin admits that for the Murngin's eight subsection system the adoption of matrilateral cross-cousin marriage is quite unlawful, but that in order to adapt this marriage system to the eight subsection system, the Murngin contrived the ingenious and practical method of prolonging the patri-cycle two-fold, that is, transforming the original 2-generation cycle into an eight subsection system with 4-generation cycle.

Lévi-Strauss (1949) also admits that the Murngin's eight subsection system is irreducible from either the Kariera's four section system or the Arunta's eight subsection system, but postulates that it is a product of compromise and accommodation of the Murngin's original four intermarrying group to an imported, full-fledged section system. He insists that there is no relationship between the prolonging of a patri-cycle and the regulation of a marriage system; thus in the diagram proposed by Elkin to depict the Murngin's eight section system, matrilateral crosscousin and patrilateral cross-cousin still fall into same section, meaning that the marriage rule regulating the diagram is bilateral cross-cousin marriage and not matrilateral cross-cousin marriage as Elkin believed. Lévi-Strauss's observations is very pertinent, and recently Barnes (1967: 17) has demonstrated it again. Judging from the fact that Elkin's diagram is composed of only two sets of patri- and/or matri- groups, that marriage regulation under the given model inevitably must be bilateral cross-cousin marriage requires no further certification.

Thus Lévi-Strauss has advanced one step in proposing a new hypothesis that: the Murngin system is originally composed of four intermarrying groups, divided between two (implied or explicit) patrilineal moieties but without matrilineal moieties, the connubial relations being unilateral (asymmetric) and a son belonging to the group of his mother's brother's wife (Josselin de Jong 1952: 17). Then through a sophisticated discussion he proposes his famous hypothesis that 'regular' and 'alternate' marriage must alternate (in male line) with the generation'. This skillfully conceived idea of two kinds of marriage pairing makes the establishment of two matri-moieties possible and finally succeeds in fusing a newly-adopted eight-subsection system into an original matrilateral cross-cousin marriage system.

The marriage rule regulating the four marriage groups designated by Lévi-Strauss as the original type of the Murngin system is uncle/niece marriage and not matrilateral cross-cousin marriage as he thought (Liu 1968). Though the concept of postulating the Murngin's eight-section system to be developed from four marriage groups is unique, owing to his misunderstanding of the basic materials, Lévi-Strauss constructs his theory on a false premise. The mystery of what kind of procedure has produced the Murngin's eight-subsection system practicing matrilateral cross-cousin marriage is not yet solved.

Concerning the third question, since Elkin and Lévi-Strauss deny the rationality of the principle of the 4-generation cycle, no anthropologists have considered this problem properly. The structural models proposed by scholars are all designed based on the principle of alternating generations or a 2-generation cycle. The model jointly proposed by Lawrence and Murdock (1949) is the only diagram adopting the marriage alliance as its theoretical base; they declare that their model is designed after the principle

of the 4-generation cycle, but in reality their diagram indicates that they follow Warner's hypothesis that 'two subsections compose one section'. This flaw leads Lawrence and Murdock to miss the opportunity to be the first to propose a correct structural model of the Murngin system.

Recently, Louis Dumont (1966) again adopts Lévi-Strauss's hypothesis to reconstruct the structural model of the Murngin system. Lévi-Strauss's hypothesis has the effect to simplifying the 4-generation cycle to a 2-generation cycle (see chapter 3), and at the same time it is also able to reorganize the eight subsections into four sets of decent groups. This not only permits the rejection of Warner's unsatisfactory postulate, but is also to fulfil the requirement that the marriage alliance needs the participation of three or more than three descent groups. If Elkin's assertion is correct, that is, that the Murngin's generation cycle is not four but two, we will not hesitate to admit that Dumont's model is the best designed and most persuasive of all the diagrams ever proposed. Dumont declares that in the past the theoretical study of the section system relied completely on the principle of 'double descent', but this does not fit the real. condition. For example, the matri-moieties of the Kariera or Arunta are purely an anthropologists' fabrication, not the product of the aborigines' thinking. The principle of alternating generations can independently solve all section problems. Thus Dumont insists that anthropologists must get out from under the spell of double descent, and the section systems constructed by the Australian aborigines must be revaluated.

The principle of alternating generations, which Dumont believes to be a real panacea, we may put aside for the time being. But when we re-examine Lévi-Strauss's hypothesis, we can not help admitting that it is a far-fetched interpretation which he provides in postulating that 'regular' and 'alternate' marriage must be practiced in alternating generations and between siblings of different sex. The realization of such a system is only possible under special conditions, and it is quite contradictory to the report that alternate marriage is merely occasionally practiced by the Murngin (Webb 1933). On the other hand, according to Dumont's model, each horde is inevitably divided to compose two descent groups, but this also contradicts the facts reported by Warner (1937: 27) who clearly denied the devision.

In a word, all theories proposed by our predecessors are constructed on the principle of alternating generations, but each model shows some discrepancies with the actual descriptions reported by field workers. The principle of alternating generations as the fundamental principle regulating the Murngin system needs thorough re-examination. The reappraisal of the principle of the 4-generation cycle discovered by Elkin and of marriage alliance explored by Dutch scholars is also required.

Beside this, the complicatedness of the Murngin's kinship terminological network also makes us feel as if we are straying into a labyrinth. Warner's report reaches seven patri-lines and five generations, Webb extends it to eight patri-lines and nine generations (Lawrence and Murdock 1949). For its imposing scale and the intricacy of the relationships, the Murngin system is really matchless. Both edges of Warner's table hang in midair, leaving a series of unsolved questions to the reader. Therefore, not a few scholars have discussed this problem (e.g., Lévi-Strauss 1949; Lawrence and Murdock 1949; Radcliffe-Brown

1951; Leach 1951; Elkin 1953; Berndt 1955; White 1963; Barnes 1967).

This problem concerns such basic structures as marriage alliances, eight subsections and moieties, which add to the complication. In the discussion of the Murngin kinship terms Barnes, after proposing a series of questions, concludes that the resources accumulated in the past are still not enough to explain the actual situation of the Murngin kinship structure, and the solution of the problem depends on the finding of new clues by field workers in futre. Barnes's conclusion is the best sketch of the present stage of the Murngin studies, and the Murngin controversy is still entangled.

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SECTION AND GENERATION CYCLE

As an initial step to the analysis of the Murngin kinship system, let us start from the clarification of the basic properties inherent in the moiety and section system. The Murngin's sixtyodd patrilineal hordes, which are the minimum unit of an exogamous group, are divided into two moieties which intermarry. These are known as Dua and Yiritcha respectively. Each moiety is further subdivided into four sections, making a total of eight sections. Each section has a proper name which is further distinguished by sex. In Table 2, following convention, masculine names are represented by capital letters, and feminine by small letters. Though the distinction has its importance in general description, it obviously only complicates discussion, so common names for both sexes are here adopted for the sake of simplicity. For this purpose the masculine name is chosen, and for distinction from the original usage, the first letter is kept in capitals and the rest is changed into lower case. The spellings of respective sections reported by the field workers do not always coincide, sometime adding to our bewilderment (see Barnes 1967:14). In this paper we adopt Webb's system. Besides this, for convenience of discussion, miscellaneous symbols used by other authors are given for the respective sections in Table 2 (limited to those discussed in this paper). Simultaneously the symbols used in this paper are also added in the last column.

TABLE 2
MOIETY, SECTION AND SYMBOL

Section Symbol WARNER WARNER ELKIN DUMONT LIU	BURALUNG A ¹ B ² Bib O	WAMUT. D1 C1 B2b P	BALLUNG (BELIN) A ² B ¹ Bia Q	KAMERDUNG D ² C ² B2a R.	BURLAIN B ² A ² Alb S	KAIDJAWK C' D' A2b T	NARIT B ¹ A ¹ Ala U
S S S S S S S S S S S S S S S S S S S	BURALANG kalian	WARMUT warmutjan	BALANG	KARMARUNG kumandjan	BULAIN bulaindjan	KAIJARK koitjan	NGARIT ngaritjan

Murngin marriage rules may be summarized by saying that a person of one section can marry into either one or both of two sections in the opposite moiety, but into none of his own moiety (Warner 1937:118). Webb reports some discrimination exists between the two, for one is regular marriage and the other is alternate or optional, only practiced under certain circumstances (1933:408). But Warner does not give any difference between the two, and admits that the Murngin's eight named divisions are grouped into pairs, so that actually two named divisions compose one unnamed section. Thus Warner insists that the Murngin system is practically formed by four implicit sections and expresses it in the following diagram:



FIG. 2. Warner's chart of the Murngin section system

Warner's theory reveals the existence of a very extraordinary descent rule. That is, the children's section is always determined by the mother's position in the eight sections, never by the father's. In other words, for a male, his children's position shifts according to the choice of spouse from either subsection; but for a female, her children always belong to the same subsection of either subsection she may marry into. In Figure 2, if a man of A¹ marries a B¹ woman, the children are D²; if the

same man marries a B^2 woman, the children are D^1 . On the contrary, if a B^1 woman has an A^1 husband, the children are D^2 ; if she has an A^2 husband, the children also are D^2 . The mother's subsection is final in determining the child's subsection. Within the Murngin subsection system no patrilineal cycles are traceable, but two matrilineal 4-generation cycles are derivable. The first is $B^1 \longrightarrow D^2 \longrightarrow B^2 \longrightarrow D^1 \longrightarrow$ and B^1 ; the second, $A^1 \longrightarrow C^1 \longrightarrow A^2 \longrightarrow C^2 \longrightarrow$ and A^1 . These two cycles are expressed by Warner by the following diagram:

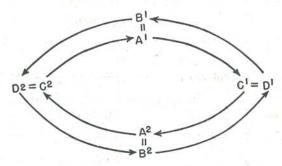


FIG. 3. Warner's matrilineal cycles

Thus Warner comes to a paradoxical conclusion stating that the kinship system of the Murngin is patrilineal and matrilineal; but the subsection system in descent, which is only an extension of the kinship system, is purely matrilineal (1937:120).

If the implicit or un-named 'four-section system' proposed by Warner as the Murngin system really exists, the significance of the Murngin's eight-subsection system or matrilateral cross-cousin marriage as its structural properties become doubtful, for excepting the Tarau case, no matrilateral cross-cousin marriage system is constructable with a four section system however skilfully marriage combinations may be arranged among them (Liu 1968). Warner's assertion of the descent rule in subsections as being purely matrilineal, also conflicts with the fact that the Murngin's hordes which are organized by subsections are actually regulated by patrilineal principles. Recently Barnes, based on Warner's diagram of matrilineal cycles as shown in Figure 3, postulates the marriage pair depicted in the Figure must be the main type chosen by Warner; he then points out that the marriage rule regulating the diagram is not that stated by Warner in the report, as the diagram only allows Arunta type second cross-cousin MMBDD marriage, and first cross-cousin MBD and FZD marriage are precluded (Barnes 1967:15).

Warner's doubtful conclusion is caused by his misunderstanding of the distinction of regular and alternate marriage and his neglect of the importance of the marriage alliance as the base of matrilateral cross-cousin marriage systems. The former mistake is corrected by Webb and Elkin in their papers. Webb, based upon his own six years' experience in Arnhem Land district, following the aborigines's actual marriage customs compiles a table to show the marriage relationships among the subsections. Webb's table is simplified and rearranged as follows:

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THE OUTCOME OF MURNGIN MARRIAGE CLASS SYSTEM

	Dua man	Yiritcha woman	Dua children
-	Buralang	Bulain	Warmut
	(Buralang)	Ngarit	Karmarung
	Warmut	Kaijark	Balang
	(Warmut)	Bangardi	Buralang
	Balang	Ngarit	Karmarung
	(Balang)	Bulain	Warmut
	Karmarung	Bangardi	Buralang
	(Karmarung)	Kaijark	Balang
	Yiritcha man	Dua woman	Yiritcha children
4	Bulain	Buralang	Kaijark
	(Bulain)	Balang	Bangardi
4.00	Kaijark	Warmut	Ngarit
	(Kaijark)	Karmarung	Bulain
	Ngarit	Balang	Bangardi
	(Ngarit)	Buralang	Kaijark
100	Bangardi (Bangardi)	Warmut	Bulain Ngarit

This table reveals many interesting structural mechanisms. Additionally to the matrilineal cycle reported by Warner, we also find the patrilineal one which is regulated by the same generation cycle, but its rule is more complicated than the previous one. If regular marriage is strictly practiced, patrilineal four-generation cycles will come out with the following sequences:

Dua:	$Buralang \longrightarrow Warmut \longrightarrow Balang \longrightarrow Karmarung$
Products	→Buralang
Yiritcha:	$Bulain \longrightarrow Kaijark \longrightarrow Ngarit \longrightarrow Bangardi \longrightarrow$
7.0	Pullain

In Australia most section systems are typically characterized by alternating generations, that is, a grand-son falls into the same section as that to which his grand-father belonged, to compose a 2-generation cycle. But in the Murngin system, a grand-son does not fall into his grand-father's section; the cycle will not be completed until a great-great-grand-son falls into his great-great-grand-father's section to close the 4-generation cycle. Elkin, basing his conclusions on this phenomenon, depicts the Murngin section system in the following diagram:



FIG. 4. Elkin's Murngin section system

In this diagram such properties as the patrilineal 4-generation cycle as well as the matrilineal ones are considered, but still such basic ones as a marriage alliance whice needs three or more than three hordes for implementing matrilateral cross-cousin marriage are ignored. The eight sections are divided into two and marriage is practiced between them, thus no matrilateral cross-cousin marriage is practicable and bilateral cross-cousin marriage is inevitably produced. The fallacy of the implied marriage regulations in the diagram has already been pointed out by Lévi-Strauss (1949:221) and Barnes (1967:16).

Contrasting with the regular marriage mentioned above, if alternate or optional marriage is also strictly practiced then the following patrilineal 4-generation cycles will appear:

Dua: Buralang→Karmarung→Balang→Warmut
→Buralang

Yiritcha: $Bulain \longrightarrow Bangardi \longrightarrow Ngarit \longrightarrow Kaijark \longrightarrow$ Bulain

Comparing this with the patrilineal cycle of regular marriage, we see that the order of circulation is just in the reverse direction. Supposing the circulation of regular marriage to be positive, then the cycle of alternate marriage is in the reverse direction or negative. Thus we may call the former a 'positive cycle' and the latter a 'negative cycle'. In a matrilineal cycle the 'generation transition' is not effected by the choice of marriage according to Warner, so we may compare this with traffic rules as a 'one-way' street and the patrilineal cycle as a 'two-way' street where regular and alternate marriages take place in reverse directions.

Judging from common sense, alternating marriages should not occur so frequently as regular ones, but Webb's report only vaguely mentions that 'under certain circumstances (they) may marry' which does not give us a clear picture of how the Murngin really use them. If we are allowed to surmise that the regular and alternate marriages should be practiced under certain strict rules, then through numerous combinations and permutations of the two marriage systems we can work out various kind of interesting descent rules.

From this point of view, Lévi-Strauss's hypothesis is one of the most dramatic postulations. The principle that 'regular and alternate marriage must alternate with the generation' is applied by the proposer himself in his diagram of a marriage alliance composed of seven patrilines (1949:229), but in the diagram of an 8-subsection system this principle breaks down and a conventional model proposed by Elkin is adopted, though some improvement is added (i. e., a sign to indicate alternate marriage is added). Furthermore, owing to the fact that the conventional diagram intrinsically shows sister-exchange marriage, Lévi-Strauss inevitably has to give the marriage restriction necessary for the matrilateral cross-cousin marriage in the text (ibid: 240–241). The application of Lévi-Strauss's hypothesis in the design of the structural model of Murngin's 8-subsection system is finally realized by Dumont in the following diagram:

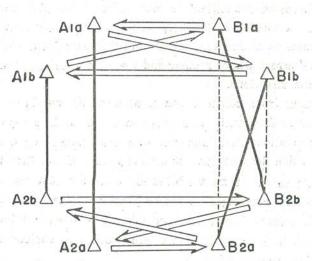


FIG. 5. Dumont's asymmetrical intermarriage in an eight-section system (one half of the hypothetical Murngin system),

The principle of both positive and negative cycles are dexterously utilized in this diagram. For example, if a Ngarit man contracts a regular marriage, his children belong to Bangardi. and in the same way if the Bangardi in turn makes a regular marriage, then his children belong to Bulain; but if the Bangardi makes an alternate marriage, then his children belong to Ngarit, returning to their grand-father's subsection. According to Lévi-Strauss's hypothesis, if the first generation (in the male line) contracts a regular marriage, the next generation must practice alternate marriage, the third generation reverts to the subsection to which the first generation belonged and the routine is repeated. If this principle is applied strictly from generation to generation, the generation cycles are inevitably limited to two subsections, in the above case, Ngarit and Bangadi. Thus the original 4generation cycle is disorganized and superseded by alternating generations or 2-generation cycle, and instead of the original two 4-generation cycles, four 2-generation cycles are produced. The four pairs proposed by Dumont are:

```
Ngarit (A1a) \longrightarrow Bangardi (A2a) \longrightarrow Ngarit

Bulain (A1b) \longrightarrow Kaijark (A2b) \longrightarrow Bulain

Balang (B1a) \longrightarrow Warmut (B2b) \longrightarrow Balang

Buralang (B1b) \longrightarrow Karmarung (B2a) \longrightarrow Buralang
```

In the past the principle of alternating generations or 2-generation cycle has been observed by anthropologists as the sole criterion for solving the intricate section systems, and its position as guidepost has never been doubted up to present. For this reason, since the phenomenon of the 4-generation cycle was discovered it has been regarded as absurd by the specialists. Those interpretations proposed by the specialists, Warner's

fictitious 4-section theory, Lévi-Strauss's hypothesis and Dumont's 8-section model as mentioned above, may be regarded as products of a kind of rationalization, hoping to explain the Murngin system by traditional principles. But, is it true that the alternating generation principle is the only one regulating the section system? Is the 4-generation cycle truly unreasonable?

Concerning these points, we should recall the facts that Warner has already discovered the existence of matrilineal 4generation cycles regulating the Murngin system and that in the well-known Arunta system, the theoretically accepted matrilineal lines are also regulated by this principle. In practice the rule of alternating generations is not the only principle regulating the section systems; the principle of the 4-generation cycle also occupies an important position. Ignorance of the factors other than that of alternating generations has led the study of the Murngin system into a theoretical blind alley and caused endless debate with impracticable models such as those mentioned above. The concept of admitting alternating generations as the only principle for analysing the section system should be abandoned, and other factors such as the principle of the 4-generation cycle should be re-evaluated. The working hypotheses proposed in this paper are designed to prevent the above-mentioned defects, and new structural models of the Murngin system will be constructed based on these new premises.

CIRCULATING CONNUBIUM

As is generally known, for the practice of unilateral cross-cousin marriage in a prescriptive marriage system, three or more than three descent groups or exogamous units are necessary for the formation of the so-called 'circulating connubium' or 'marriage alliance'. But we are not well informed of the actual way in which the marriage alliance is organized when the given society possesses numerous exogamous units. Rodney Needham's analysis of the Eastern Sumba's circulating connubium presents one of the best examples (Needham 1957).

According to Needham, the marriage system of the Eastern Sumba is composed of 24 exogamous units, forming 12 marriage alliances. The number of units organizing a marriage alliance ranges from 3 to 7, the major numbers being 4, 5 and 6. Among the 24 exogamous units, over half participate in a single marriage alliance only, but nevertheless some units participate in several. The maximum number of perticipation is 8, with four units belonging to this category. The other numbers of participation are 6, 5, 4 and 2, with one unit belonging to each category respectively. In other words, the Eastern Sumba's marriage alliance is not an elaborate single one in which all units participate, but an assemblage of many small alliances.

The only materials available for the study of the Murngin marriage alliance is White's map illustrating the marriage re-

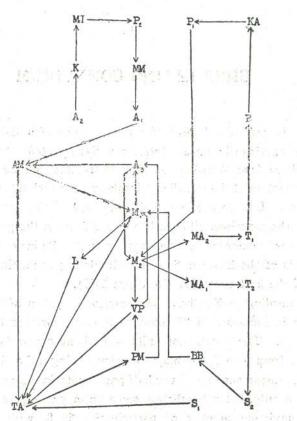


FIG. 6. Needham's diagram of circulating connubium in Eastern Sumba.

lationship among the hordes or exogamous units mentioned in Warner's report (White 1963:123).

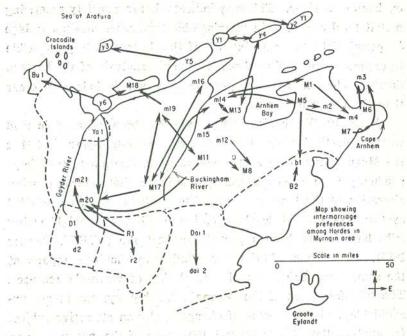


FIG. 7. White's map of circulating connubium in east Arnhem Land.

On this map only the incomplete marriage relationships of 36 Murngin hordes are represented. Though not a single complete marriage alliance is depicted, this does not prevent us from recognizing the basic properties involved. There is no indication of an overall organization into one single marriage alliance, but on the contrary the hordes appear to be divisible into several groups. This phenomenon is quite similar to the Eastern Sumba case. Meanwhile, this map reveals some discrepancies with the Eastern sumba. For example, the Murngin diagram indicates the goincidental existence of bilateral cross-cousin marriage or sister-

exchange marriage. This may indicate that in a society practicing matrilateral cross-cousin marriage bilateral cross-cousin marriage is permissible to same extent. If this is true, it is permissible to express some doubts about Needham's analysis of the Eastern Sumba, wherein marriage practice does not include any other than the prescribed ones.

Recently, W. Shapiro (1967), who has been engaged in field work in north Arnhem Land, reports the interesting fact that the Murngin marriage system shows an inclination to transform gradually from a matrilateral system into a bilateral one, causing alteration of the kinship terms. The abolishment of an original system and the wholesale adoption of that of a neighbouring tribe has already been reported for Australia (W. E. H. Stanner 1933). This lends additional credibility to Shapiro's report, at the same time enhancing the reliability of Warner's resouces. The transformation of the Murngin kinship system seems unavoidable, and the process of change is also an attractive subject for study. But this is beyond the scope of the present paper. Here the Murngin marriage system will be treated in its classic form characterized by matrilateral cross-cousin marriage as first observed and reported by Warner forty years ago.

For societies genuinely regulated by unilineal descent rules, whether they are patri- or matri-lineal, the basic structures—totally or partially—are expressible by genealogical trees such as shown in Figure 8a and 9a. In these Figures the solid line represents the father-child link and the dotted line the mother-child link. Now the 'genealogical tree' is divisible into two, a 'patri-tree' and a 'matri-tree', the former representing a patrilineal descent group and the latter a matrilineal descent group. If in

a given society the lineality and collaterality are ignored and members of the same generation are identified, together, then the genealogical tree is simplified by the adoption of a line by which each generation is represented by two units, m (male) and f (female). We may call this simplified line a 'patri-line' or 'matri-line'. (See Figures 8b and 9b.) If we ignore the sex, further more, then each generation of the genealogical line can be expressed by a simplified unit—sex variable 'a' (see Figure 8c and 9c). If a distinction is necessary between the two lines, we may call the former a 'sex-distinguishing patri- or matri-line'. Each unit of the generation of a genealogical line is called a 'segment'.

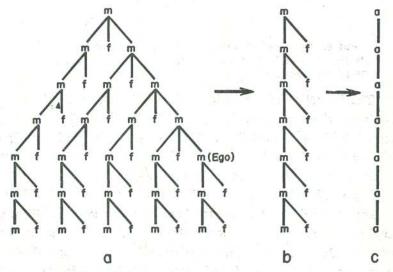


FIG. 8. Genealogical tree: patrilineal. Lines represent fatherchild link. a. Patri-tree. b. Patri-line, sex-distinguished. c. Patri-line, sex-ignored.

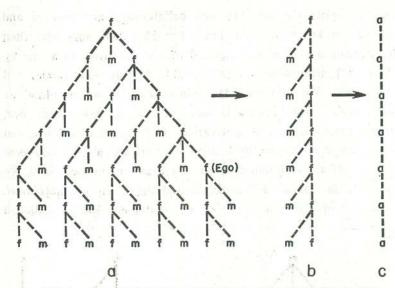
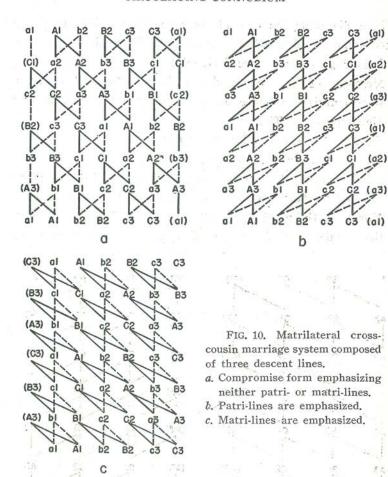


FIG. 9. Genealogical tree: matrilineal. Lines represent mother-child link. a. Matri-tree. b. Matri-line, sex-distinguished. c. Matri-line, sex-ignored.

The formation of a marriage alliance by matrilateral crosscousin marriage is expressible by genealogical lines. The simplest one needs three descent lines. This simplest form of marriage alliance is employed for the study of the mathematical properties of the matrilateral cross-cousin marriage system.

Figure 10a adopts three sex-distinguishing genealogical lines to express the simplest matrilateral cross-cousin marriage alliance, emphasizing neither parti- nor matri-linealities. By tracing the solid and dotted lines, we may easily find the marriage alliance is composed of three patri- and three matri-lines. In the given diagram patri-lines are represented by A, B, C and matri-lines by 1, 2 and 3. In Figures 10b and 10c the patri- and matri-lines



are emphasized respectively. If sex-ignoring genealogical lines are adopted, the diagram is expressible by a more concise form. This is shown in Figure 11, where 11a represents patri-line emphasized and 11b matri-line emphasized marriage alliances.

In these diagrams the descent lines are extendable limitlessly to ascending and descending generations. From these diagrams the following properties are derivable:

(1) Members of these three descent-lines associated with the same marriage alliance are categorized into the following nine segments. They are:

A1 A2 A3 B1 B2 B3 C1 C2 C3

(2) These segments are produced by the intersection of three patri-lines and three matri-lines, so the sum of the segments is equal to the product of the numbers of both descent lines. Moreover, the numbers of patri- and matri-lines are equal to the numbers of descent groups organizing the marriage

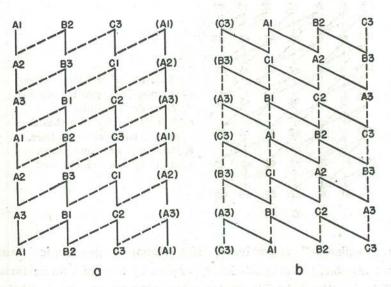


FIG. 11. Matrilateral cross-cousin marriage system composed of three descent lines in simplified form.

alliance. Thus, if the marriage alliance is derived from n descent groups (n is 3 or larger than 3), the *number of the segments Ns* is deduced by the following formula:

$$Ns=n^2$$
 $n \ge 3$

Each patri- and matri-line consists equally of three segments and performs regular circulations. This system is regulated by 'the principle of a 3-generation cycle'. The cycles of three patrilines are:

$$\begin{array}{ccccc} A1 & \longrightarrow & A2 & \longrightarrow & A3 & \longrightarrow & A1 \\ B2 & \longrightarrow & B3 & \longrightarrow & B1 & \longrightarrow & B2 \\ C3 & \longrightarrow & C1 & \longrightarrow & C2 & \longrightarrow & C3 \end{array}$$

The other three matri-cycles are:

Perceiving the fact that each descent line is expressible by a limited number of segments (here the number is three), and that the marriage alliance consists of nine segments, the model of the system is constructable by these limited elements. This is shown in Figure 12, where the three patri-lines are more emphasized than the matri-lines. Bases on this diagram the mathematical properties of the matrilateral cross-cousin marriage system will be discussed.

In Figure 12, we adopt two generators m and f, the former represents the father-child link by a solid line, and the latter represents the mother-child link by a dotted line as shown in the previous Chapters. The arrow-head indicates the transition of generations from ascending to descending. The number of the generation is expressed by the exponent of the generators. In

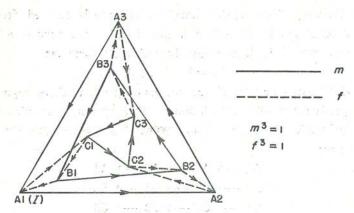


FIG. 12. Structural model of matrilateral cross-cousin marriage system composed of three descent lines. m: father-child link f: mother-child link

this case the positive number stands for ascending generations and the negative number for the descending. For example, m^2 stands for father's father's generation; f^{-2} stands for sister's daughter's daughter's generation. Thus the null number, m^0 or f^0 , inevitably represents no ascending or descending generation, that is, Ego's generation (I). The exponents of the same descentline are computable; for example,

$$m^2 \cdot m^{-1} = m^{2+(-1)} = m^{2-1} = m$$
.

Suppose Ego is placed in segment A1, say, A1=I, then starting from this point and following the patri-line in the direction of the arrow-head, the following equations can be deduced:

A1=
$$m^0$$
, A2= m^{-1} , A3= m^{-2} , A1= m^{-3} , A2= m^{-4} , A3= m^{-6} , A1= m^{-6} , A2= m^{-7} , A3= m^{-8} ,

In this case every descent line is regulated by the principle of a 3-generation cycle, that means the exponent is determined by

modulo 3 or $m^{-3}=I$. By the application of this rule the abovelisted equations can be rewritten as follows:

A1=
$$I$$
, A2= m^{-1} , A3= m^{-2} , A1= I , A2= m^{-1} , A3= m^{-2} , A1= I ,

Now we convert the transition in the direction of the ascending generation, the ruling principle becomes $m^3=I$, and each segment can be expressed as follows:

A1=I, A3=m, A2= m^2 , A1=I, A3=m, A2= m^2 , Synthesizing the above two cases, the ruling principle is expressible by $m^{\pm 3}$ =I and each segment is tabulated as follows:

A1=
$$I$$
 A2= $m^{\pm 3-1}$ A3= $m^{\pm 3-2}$

For the same reason, the matri-line passing through A1 is regulated by the principle of $f^{\pm 8}=I$, and its segments are expressible by the following equation:

A1=
$$I$$
 C1= $f^{\pm 3-1}$ B1= $f^{\pm 3-2}$

The paths to the other segments are numerous, for example, the approach to C3 is:

$$C3=f^2m$$
, mf^2 , $f^{-1}m^{-2}$, $m^{-2}f^{-1}$,

Among then the form $f^{\pm 3-n}m^{\pm 3-n}$ is chosen as a representative for each segment. The remaining four segments are expressed as follows:

B2=
$$f^{\pm 3-2}m^{\pm 3-1}$$
 B3= $f^{\pm 3-2}m^{\pm 3-2}$
C2= $f^{\pm 3-1}m^{\pm 3-1}$ C3= $f^{\pm 3-1}m^{\pm 3-2}$

Conbining the above-mentioned equations, we find that each segment is replacible by the multiplication of two other segments. For example, if $B2=f^{\pm 3-2}m^{\pm 3-1}$ is replaced by $B1=f^{\pm 3-2}$ and $A2=m^{\pm 3-1}$, the following equation can be deduced:

$$B2 = B1 \cdot A2$$

Thus the multiplication of any two segments will result in one

of the given segments. The product of the multiplication of these nine segments is shown as follows:

TABLE 4
9-SEGMENT MULTIPLICATION TABLE

inen•ge i	A1	A2	A3	B1	B2	B3	C1	C2	C3	- 1
A1	A1	A2	A3	В1	B2	В3	C1	C2	СЗ	2.
A2	A2	A3	A1	B2	B3	BI	C2	C3	C1	
A3	A3	A1	A2	B3	B1	B2	C3	C1	C2	
: B1	B1	B2	B3	C1	C2	C3	A1	A2	A3	1
B2	B2	B3	Bí	C2	C3	C1	A2	A3	A1	
B3	B3	B1	B2	C3	C1	C2	A3	A1	A2	T
C1	C1	C2	C3	A1	A2 ·	A3	BI	B2	B3	
C2	C2	C3	C1	A2	A3	A1	B2	B3	B1	
C3	C3	C1	C2	- A3	A1	A2	B3	B1	B2	
					-					

The table reveals that:

(1) The marriage alliance of the matrilateral cross-cousin marriage system consisting of three descent lines (G₃²: superscripts indicate the generation cycle and subscripts indicate the number of descent groups) is expressed by a set of nine segments as its elements as shown in the following diagram.

$$G_3^3 = \begin{cases} A1 & C1 & B1 \\ A2 & C2 & B2 \\ A3 & C3 & B3 \end{cases} \qquad \begin{array}{c} m: \text{ mod} = 3 \\ f: \text{ mod} = 3 \end{array}$$

- (2) 'Generation transition' is a binary operation with the elements.
- (3) The set contains I (A1 in this case) as an element and, for any element u, $u \cdot I = I \cdot u = u$. I is the identity element or unit element.

(4) Since I occurs precisely once in each row and column, the axiom in inverse is satisfied. We can determine at once the inverse of any group element from the table. For example, the configuration



shows us that $(B2)^{-1} = C3$ or $(C3)^{-1} = B2$.

(5) The group is commutative. For example:

$$B2 \cdot C2 = C2 \cdot B2 = A3.$$

- (6) The rows and columns are permutations or rearrangements of the elements in the top row and left column, respectively —the 'coincidence' previously observed.
- (7) The 3×3 square in the upper left-hand corner is precisely the multiplication table of generator m. If M represents the set of elements in this upper left-hand square, the Table 5 symbolizes this pattern-within-a-pattern in the multiplication table.

TABLE 5
SIMPLIFIED PATTERN OF THE MULTIPLICATION TABLE

M	B1 • M	C1 · M
B1 • M	C1 · M	M
C1 · M	M	B1 · M

(8) Associatively: This property is satisfied as $(A2 \cdot B3) \cdot C1 = A2 \cdot (B3 \cdot C1) = A1.$

In dealing with group elements and their relations, it becomes necessary to answer the following question: If a and b are known elements of a group, is there an element x of the group such that ax=b? We claim that $x=a^{-1}b$ is the group element we seek, for

$$a(a^{-1}b) = (aa^{-1})b = Ib = b.$$

that is, $x=a^{-1}b$ satisfies the group 'equation' ax=b. Applying this to kinship problem, for such question as $C2 \cdot x = B1$ (What are my B1 to my C2?) The answer is

$$x = (C2)^{-1} \cdot B1 = B3 \cdot B1 = C3.$$

Utilizing the multiplication table, we also can find the answer immediately. The configuration



shows us that C2·C3=B1, C3 is the answer.

In the above description, we have discussed the mathematical properties of the marriage alliance of a matrilateral cross-cousin marriage system which is organized by three descent groups. We see that the nine elements are performing a high mathematical function. But, owing to the adoption of a non-numerical system for their symbols, they are prevented from a direct mathematical manipulation. No binary operation is possible without the help of a multiplication table. It would be ideal if a system of a set of numerical notations were explored which fits the operation.

For this purpose, the exponents of generators are most suitable resources for the construction of a numerical notation system.

We have chosen the form $f^{\pm 3-n}m^{\pm 3-n}$ as a representative for each segment. Now we adopt a two place system and assign the first place to the n value of the exponent of f and the second place to the n value of the exponent of m. Thus, for example, $C3=f^{\pm 3-1}m^{\pm 3-2}$ is represented as 12. But this is hardly distinguishable from the numerical notation system for the 'kinship category' (Harvey and Liu 1967). So in this case 'S' is added in front for discrimination. C3 is represented by S12 instead of 12. Now group G_3^3 can be expressed as a set of 9 elements in the following diagram:

$$G_8^3 = \begin{cases} S00 \text{ (A1)} & S10 \text{ (C1)} & S20 \text{ (B1)} \\ S01 \text{ (A2)} & S11 \text{ (C2)} & S21 \text{ (B2)} \\ S02 \text{ (A3)} & S12 \text{ (C3)} & S22 \text{ (B3)} \end{cases} \qquad \begin{array}{c} \textit{m: mod=3} \\ \textit{f: mod=3} \\ \end{cases}$$

The mathematical properties of G3 are to be treated as follows:

(1) Binary operation: The generations of the same descent line (or the exponents of the same generators) are computable. The formula is

$$Sx_1y_1 \cdot Sx_2y_2 = S(x_1+x_2)(y_1+y_2)$$

For example, $S10 \cdot S21 = S(1+2)(0+1) = S01$.

(2) Identity: S00 is the identity segment. For example

$$S00 \cdot S21 = S21 \cdot S00 = S21.$$

(3) Associativity: This property is satisfied as

$$(\mathbf{S21} \cdot \mathbf{S11}) \cdot \mathbf{S02} = \mathbf{S21} \cdot (\mathbf{S11} \cdot \mathbf{S02}) = \mathbf{S01}.$$

(4) Inverses: Each element (or segment) has its inverse in the set. The formula is

$$(Sxy)^{-1} = S(3-x)(3-y)$$

For example, $(S21)^{-1} = S(3-2)(3-1) = S12$.

(5) Commutative: This property is shown as

$$S12 \cdot S02 = S02 \cdot S12 = S11.$$

Because it is commutative, and because of the finite number of its elements, this group is called a 'finite Abelian group'. The multiplication table is rearranged as follows:

TABLE 6
MULTIPLICATION TABLE IN NUMERICAL
NOTATION SYSTEM

	•	S00	S01	S02	S10	S11	S12	S20	S21	S22	
	S00	S00	S01	S02	S10	S11	S12	S20	S21	S22	_
	S01	S01	S02	S00	S11	S12	S10	S21	S22	S20	
*	S02	S02	S00	S01	S12	S10	S11	S22	S20	S21	
	S10	S10	S11	S12	S20	S21	S22	S00	S01	S02	
	S11	S11	S12	S10	S21	S22	S20	S01	S02	S00	
	S12	S12	S10	S11	S22	S20	S21	S02	S00	S01	
	S20	S20	S21	S22	S00	S01	S02	S10	S11	S12	
	S21	S21	S22	S20	S01	S02	S00	S11	S12	S10	
	S22	S22	S20	S21	S02	S00	S01	S12	S10	S11	

STRUCTURAL MODEL OF THE MURNGIN SYSTEM

We have devoted the previous chapter to the mathematical study of the matrilateral cross-cousin marriage system which presents a marriage alliance in which, in its simplest form, three descent groups or hordes participate. The result proves that the working hypothesis (1) we proposed in the first chapter is applicable, and substantially the nine segments of the given system form the nine elements of a set of an 'Abelian group'. Here we take a further step and apply the hypothesis tentatively to the controversial Murngin system in order to test its applicability and determine whether any modification is required by the special properties which the Murngin system inherently possesses.

Concerning the actual condition of the Murngin marriage alliance, no systematic or reliable data are available to clarify it, so we remain ignorant of its structure up to the present. In the following, we partially rely on White's imperfect map which indicates the marriage alliances between the hordes in north-east Arnhem Land, and partially on theoretical conjecture, to establish the structural model of the Murngin system. As already mentioned, bilateral cross-cousin marriage is gradually increasing in the Murngin society today, this fact will not be considered here; we only examine the kinship system from the

standpoint of the time when matrilateral cross-cousin marriage was strictly observed.

In their vast teritory the Murngin are divided into some 60 hordes, but there is no sign of an overall organization into one single marriage alliance. On the contrary, White's map indicates that the hordes are divisible into several groups which form marriage alliances, though none of them completes their cycles. The majority of the hordes seem to join only one marriage alliance, a few join several; in general the number of hordes participating in a marriage alliance seems not very large, usually between four and eight.

This presumption, based on White's map, agrees with the condition among the Eastern Sumba. But the Eastern Sumba lack the moiety and subsection system of the Murngin. It is not hard to presume that these properties would have some determinant influences on the formation of the structural model of the Murngin system. Whether the structural model proposed in the previous chapter fits the Murngin system will be examined first.

For convenience two symbols are adopted, X and Y, the former for the Dua moiety, the latter for the Yiritcha moiety; each horde is distinguished by the symbol of its moiety, to which a number is added at the right lower corner, for example, X_1 , X_2 , X_3 ...or Y_1 , Y_2 , Y_3 Figure 11a is simplified and reproduced here as Figure 13a, which is formed by three patri-lines A, B and C. If A is assigned to X, it is required by the marriage regulation of the Murngin system that A's wife-giver B and wife-taker C belong to the opposite moiety Y, as Y_1 and Y_2 respectively. But under the mechanism of a marriage alliance

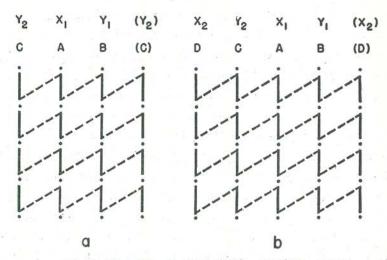


FIG. 13. Matrilateral cross-cousin marriage system.

a. Composed of three patri-lines.

b. Composed of four patri-lines.

formed by three hordes, Y_1 becomes Y_2 's wife-taker and Y_2 becomes Y_1 's wife-giver. Thus a marriage is established between the two, Y_1 and Y_2 . This is prohibited by the rule which forbids the members of the same moiety to marry each other.

From the above it is inferred that: in a marriage alliance in which three hordes participate, however we arrange their membership, no agreement with the existing regulations can be achieved. A marriage alliance formed by three hordes is not allowed within the moiety system.

In Figure 13b a marriage alliance formed by four hordes A, B, C, D, is represented. Here A is assigned to X_1 , B to Y_1 , C to Y_2 , D to X_2 . X_1 's wife-giver is Y_1 , and wife-taker is Y_2 , which agrees with the requirement that marriage should take place

between opposite moieties. In the same way Y_1 's marriage pairs, following Radcliffe-Brown's terminology, are X_1 and X_2 ; Y_2 's marriage pairs are X_1 and X_2 ; X_2 's marriage pairs are also Y_1 and Y_2 , all fulfilling the requirements. At the same time no marriage is concluded between members of the same moiety, as X_1 to X_2 or Y_1 to Y_2 . Thus, a marriage alliance formed by four hordes agree with the Murngin marriage law.

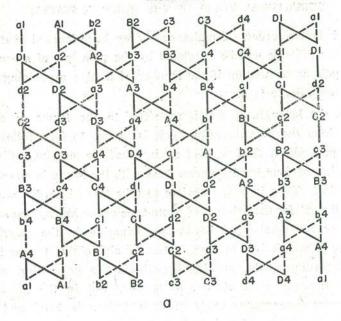
If analysis proceeds in this way, it is not difficult to find that marriage regulations prohibiting marriage between members of the same moiety are inevitably violated if the marriage alliance is composed of an odd number of hordes. The rule is observed only if the number is even. According to this we advance the following presumption: The number of hordes required to organize the marriage alliance must be even, the minimum number being four.

Another characteristic of the Murngin system is the adoption of an eight subsection system. These are, as mentioned in Chapter 3, Buralang (O), Warmut (P), Balang (Q) and Karmarung (R) for Dua; and Bulain (S), Kaijark (T), Ngarit (U) and Bangardi (V) for Yiritcha moiety. (In parenthesis are given the symbols used in this paper for the respective sections, see Table 2). The generation transition among the sections is strictly regulated by the principle of a 4-gneration cycle or circulation. The order of circulation as shown in the previous chapter, is constant for the matri-cycle and not affected by the kind of marriage, 'regular' or 'alternate'. Contrarily, the patri-cycle is flexible, the order of circulation of alternate marriage is just in the reverse direction of that of the regular ones which follows

an alphabetical order. Simultaneously we have learned that the marriage alliance will be regulated by the principle of n-generation cycle or circulation if the number of hordes organizing the marriage alliance is n.

As the Murngin system is restricted to the frame of eight subsections and its generation cycle is limited to four, following the hypothesis, it goes without saying that the marriage alliance consisting of four hordes agrees best with the Murngin marriage regulation. Thus it is appropriate to take this (4-hordes marriage alliance) for the original or proto-type of the Murngin system. However, the number of hordes participating in the marriage alliance is not limited to four, it may be six, eight, ten and so on. According to the working hypothesis the generation cycle should be equal to the number of hordes. But in the Murngin system the 4-generation cycle of each hordes is fixed and invariable regardless of the number of hordes participating in the marriage alliance. This is not fully consistent with our hypothesis, which must then be revised to make it consistent with the properties of the Murngin system.

Figure 14 shows general diagrams of prescribed matrilateral cross-cousin marriage organized by four patri-lines or hordes. Figure 14a adopts sex-distinguished genealogical lines, Figure 14b adopts sex-ignored patri-lines. In these diagrams the lines are extendable in both directions for a chosen number of ascending and desending generations. Simultaneously, owing to the property of the regulation of a 4-generation cycle, the lines are also expressible by limited length as shown in Figure 14c. As this diagram is designed as the model for matrilateral cross-cousin marriage in general, it can not be looked upon as repre-



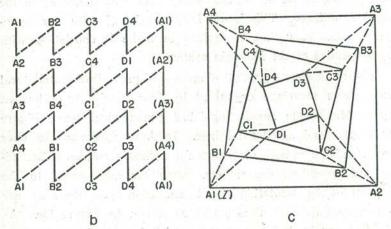


FIG. 14. Matrilateral cross-cousin marriage system composed of four descent lines. a. Generalized form. b. Simplified, patri-lines are emphasized. c. Structual model.

senting the Murngin system, for which it would need some modification and adjustment.

First, the general symbols used in Figure 14c must be replaced by the specific section symbols that are proposed for the Murngin system. Al is assigned to O_1 and Ego (I) remaining constantly O_1 , the sixteen segments shown in Figure 14c and their equivalent subsection symbols are shown in the following table.

TABLE 7
SECTIONS AND SYMBOLS

Dua	Bura	alang	War	mut	Bal	ang	Karn	arung
X ₁ X ₂	O ₁ O ₂	A ₁ C ₃	P ₁	A ₂ C ₄	Q_1 Q_2	A ₃ C ₁	R ₁	A ₄ C ₂
Yiritcha	Bu	lain	Kai	jark	Nga	rit	Bang	gardi
Y ₁ Y ₂	S ₁	B_2 D_4	T ₁	B ₃	U ₁ U ₂	B ₄	V ₁	B ₁

Now Figure 14a is replaced by Figure 15, where the subsection to which Ego (I) is subordinated is placed in the centre. Each patri-line is extended four generations ascending and descending from Ego's generation. Based this diagram two models of the Murngin system are produced in Figure 16. Of the two the first one (Fig. 16a) is especially designed to emphasize the patrilineality by which the given society is characterized. Two lines belonging to the same moiety approach each other, with the Dua (X) occupying the outside and the Yiritcha (Y)

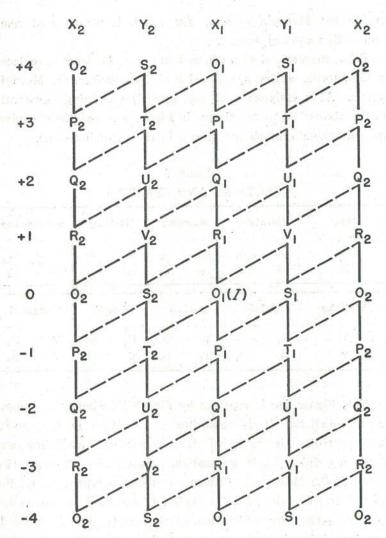


FIG. 15. Matrilateral cross-cousin marriage in Murngin system composed of four hordes.

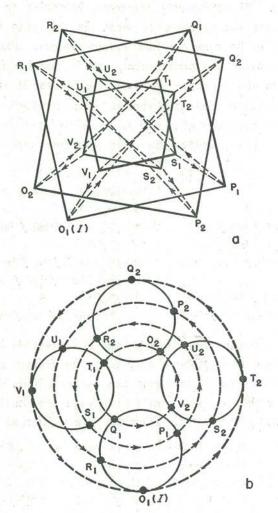


FIG. 16. Alternate structural models for Murngin system composed of four hordes.

the inside. Simultaneously segments belonging to the same subsection are also arranged to meet. In this case the matrilines have to be squeezed into narrow spaces. Figure 16b is designed to avoid this predicament, and is suitable for the representation of complicated systems (see Figures 18 and 20).

Applying the generator theory, each generation cycle is regulated by modulo 4 of m and f, or $m^{\pm 4}=I$ and $f^{\pm 4}=I$. The 16 segments composed of the four patri-lines are shown as follows:

Each segment plays multiple roles and is expressible in various forms. Those shown above only adopt generators taking the form of f^nm^n , the other forms are excluded here. Moreover, those forms shown above can also be expressed in the generalized form of $f^{\pm 4-n}$ $m^{\pm 4-n}$. They can be rewritten as follows:

Adopting the numerical notation system for a section system as proposed in the previous chapter, the Murngin's simplest marri-

age alliance G₄, can be expressed as a set of 16 segments in the following diagram.

$$G_4^4 = \begin{cases} \mathbf{S00}(\mathrm{O_1}) & \mathbf{S10}(\mathrm{T_2}) & \mathbf{S20}(\mathrm{Q_2}) & \mathbf{S30}(\mathrm{V_1}) \\ \mathbf{S01}(\mathrm{P_1}) & \mathbf{S11}(\mathrm{U_2}) & \mathbf{S21}(\mathrm{R_2}) & \mathbf{S31}(\mathrm{S_1}) \\ \mathbf{S02}(\mathrm{Q_1}) & \mathbf{S12}(\mathrm{V_2}) & \mathbf{S22}(\mathrm{O_2}) & \mathbf{S32}(\mathrm{T_1}) \\ \mathbf{S03}(\mathrm{R_1}) & \mathbf{S13}(\mathrm{S_2}) & \mathbf{S23}(\mathrm{P_2}) & \mathbf{S33}(\mathrm{U_1}) \end{cases}$$

$$m \colon \bmod{=} 4 \quad f \colon \bmod{=} 4$$

The mathematical properties characteristic for this group are shown as follows:

(1) Binary operation: The numerical numbers belonging to the same place are computable. The formula is

$$Sx_1y_1 \cdot Sx_2y_2 = S(x_1+x_2)(y_1+y_2)$$

For example: $S23 \cdot S31 = S(2+3)(3+1) = S10$.

(2) Identity: S00 is the identity segment. For example

 $S00 \cdot S02 = S02 \cdot S00 = S02.$

(3) Associativity: This property is satisfied as

$$(S21 \cdot S13) \cdot S02 = S21 \cdot (S13 \cdot S02) = S32.$$

(4) Inverses: Each element or segment has its inverse in the set. The formula is

$$(Sxy)^{-1} = S(4-x)(4-y)$$

For example $(S21)^{-1} = S(4-2)(4-1) = S23$.

(5) Commutative: This property is shown as

$$S13 \cdot S22 = S22 \cdot S13 = S31.$$

In the segment diagram each column represent a patri-line, each row a matri-line. The multiplication table of this group is shown in Table 8.

TABLE 8
16-ELEMENT MULTIPLICATION TABLE

•	800	201	S02	803	SIO	SII	S12	813	820	S21	S22	823	830	831	832	833
800	200	801	S02	803	810	811	S12	SI3	820	821	S22	S23	830	831	832	833
108	S01	802	803	800	SII	S12	813	810	S21	S22	823	820	831	832	833	830
S02	S02	803	800	801	S12	SI3	810	SII	S22	S23	820	S21	832	833	830	S31
803	803	800	201	S02	SI3	810	SII	S12	823	820	S21	S22	833	830	831	832
810	810	811	812	S13	820	S21	S22	823	830	831	832	833	830	108	S02	803
SII	SII	812	SI3	810	S21	S22	S23	820	831	832	833	830	201	S02	803	008
S12	S12	813	810	SII	S22	S23	820	S21	832	833	830	831	202	803	800	108
S13	SI3	810	SII	S12	S23	820	S21	S22	833	830	S31	S32	S03	800	801	205
820	820	S21	S22	S23	830	S31	S32	833	800	201	208	803	810	SII	S12	SI3
821	S21	822	S23	820	S31	S32	833	830	108	202	803	800	SII	S12	SI3	810
S22	S22	S23	820	S21	S32	833	830	831	208	803	008	201	S12	SI3	810	SII
S23	823	820	821	S22	833	830	831	832	803	800	108	202	813	810	SII	S12
830	830	831	832	833	200	108	202	803	810	S11	S12	S13	820	821	S22	S23
831	831	S32	833	830	201	202	803	800	SII	S12	813	810	821	S22	823	820
832	832	833	830	831	202	803	800	201	812	SI3	810	SII	S22	823	820	S21
833	833	830	831	S32	803	800	108	S02	SI3	810	SII	S12	S23	820	S21	S22
•																

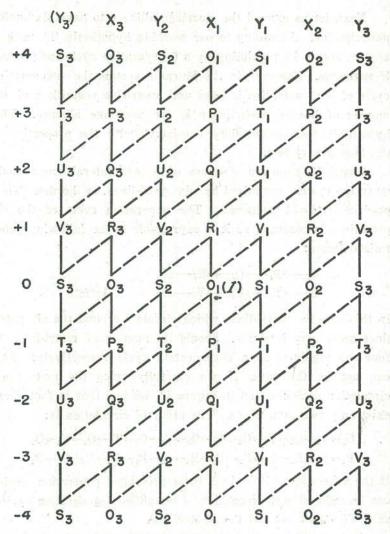


FIG. 17. Matrilateral cross-cousin marriage in Murngin system composed of six hordes.

Next, let us expand the marriage alliance to have six hordes participating. According to our working hypothesis (1), such a system should be regulating by a 6-generation cycle and produce 6² segments. However, in the Murngin system the 4-generation cycle of each patri-line is fixed and invariable regardless of the number of hordes participting is the marriage alliance. This hypothesis, then, is not fully consistent with the properties of the Murngin system.

Figure 17 shows a diagram of a matrilateral cross-cousin marriage system organized by six patri-lines or hordes. They produce $4\times6=24$ segments. The generation cycle of the six patri-lines is constant, so it is expressible in the following generalized forms:

$$\begin{array}{ll} \mathbf{O}_{i} \longrightarrow \mathbf{P}_{i} \longrightarrow \mathbf{Q}_{i} \longrightarrow \mathbf{R}_{i} \longrightarrow \mathbf{O}_{i} \\ \mathbf{S}_{i} \longrightarrow \mathbf{T}_{i} \longrightarrow \mathbf{U}_{i} \longrightarrow \mathbf{V}_{i} \longrightarrow \mathbf{S}_{i} \end{array}$$

$$1 \leq i \leq 3$$

In this case the matri-lines which circulate around the six patrilines seem very intricate. Should the number of matri-lines be four and regulated by a 6-generation cycle respectively? This can not be the case. If we carefully trace the matri-line's circulation demonstrated in Figure 17, we find that in fact there exist only two matri-lines. The order of circulation is:

$$O_{1} \rightarrow T_{2} \rightarrow Q_{3} \rightarrow V_{3} \rightarrow O_{2} \rightarrow T_{1} \rightarrow Q_{1} \rightarrow V_{2} \rightarrow O_{3} \rightarrow T_{3} \rightarrow Q_{2} \rightarrow V_{1} \rightarrow O_{1}$$

$$P_{1} \rightarrow U_{2} \rightarrow R_{3} \rightarrow S_{3} \rightarrow P_{2} \rightarrow U_{1} \rightarrow R_{1} \rightarrow S_{2} \rightarrow P_{3} \rightarrow U_{3} \rightarrow R_{2} \rightarrow S_{1} \rightarrow P_{1}$$

If the subscription is ignired, these prolonged generation cycles can be reduced to the repetition of simplified 4-generation cycles, as $O \rightarrow T \rightarrow Q \rightarrow V \rightarrow O$ and $P \rightarrow U \rightarrow R \rightarrow S \rightarrow P$.

Applying the generator theory, the two generation cycles of this group are expressible by $m^{\pm 4}=I$ and $f^{\pm 12}=I$ respectively.

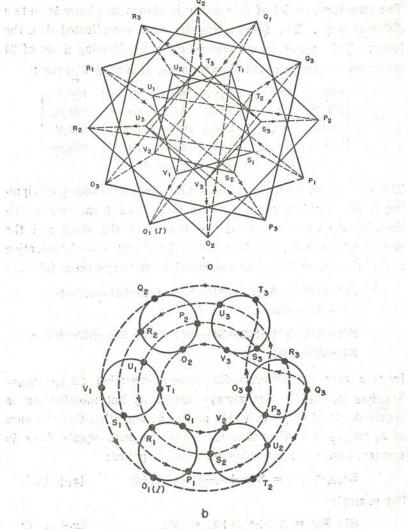


FIG. 18. Structural models for Murngin system composed of six hordes.

The structural model of this group is shown in Figure 18 in two different ways. The former appears more complicated than the latter. This group is represented as G_6^4 , forming a set of 24 segments or elements and shown on the following diagram:

$$G_6^4 = \begin{cases} S00(O_1) & S10(T_2) & S20(Q_3) & S30(V_3) & S40(O_2) & S50(T_1) \\ S01(P_1) & S11(U_2) & S21(R_3) & S31(S_3) & S41(P_2) & S51(U_1) \\ S02(Q_1) & S12(V_2) & S22(O_3) & S32(T_3) & S42(Q_2) & S52(V_1) \\ S03(R_1) & S13(S_2) & S23(P_3) & S33(U_3) & S43(R_2) & S53(S_1) \end{cases}$$

$$m \colon \bmod{=4} \qquad f \colon \bmod{=12}$$

The six colums represent the six patri-lines or hordes participating in the marriage alliance. The four rows form two matrilines by the combination of the first with the third and the second with the fourth. The order of the matri-lines' generation cycle is expressible by the numerical notation system as follows:

$$\begin{array}{l} S00 \rightarrow S10 \rightarrow S20 \rightarrow S30 \rightarrow S40 \rightarrow S50 \rightarrow S02 \rightarrow S12 \rightarrow S22 \rightarrow S32 \rightarrow \\ S42 \rightarrow S52 \rightarrow S00 \\ S01 \rightarrow S11 \rightarrow S21 \rightarrow S31 \rightarrow S41 \rightarrow S51 \rightarrow S03 \rightarrow S13 \rightarrow S23 \rightarrow S33 \rightarrow \\ S43 \rightarrow S53 \rightarrow S01 \end{array}$$

In this case the formula for binary operation $Sx_1y_1 \cdot Sx_2y_2 = S(x_1+x_2)(y_1+y_2)$ is not always adoptable, but modification is required. It is only applicable under the condition that the sum of x_1 and x_2 is smaller than 6. When the sum equals 6 or is greater than 6, the following formula is applied:

$$Sx_1y_1 \cdot Sx_2y_2 = S(x_1+x_2-6)(y_1+y_2+2)$$
 $(x_1+x_2 \ge 6)$

For example:

$$S13 \cdot S42 = S(1+4)(3+2) = S51$$
 $(x_1 + x_2 < 6)$

$$S41 \cdot S52 = S(4+5-6)(1+2+2) = S31 \qquad (x_1+x_2 \ge 6)$$

TABLE 9 24-ELEMENT MULTIPLICATION TABLE

•	800	S01	202	S03	810	S11	S12	SI3	820	S21	822	S23	830	831	832	833	S40	S41	845	843	S20	S51	S52	853	
800	800	801	S02	S03	810	S11	S12	S13	820	S21	S22	S23	830	831	832	833	840	SH	842	SIS	820	S51	S52	S53	1
108	801	S02	803	800	SII	S12	SI3	810	S21	\$22	823	\$20	S31	832	833	830	841	842	843	S40	S51	S52	853	850	
202	S02	803	800	201	S12	SI3	810	SII	\$22	\$23	820	S21	832	833	830	831	842	843	840	HS	S52	S53	850	S51	
803	803	800	801	S02	SI3	810	SII	S12	S23	820	S21	822	833	830	831	832	843	078	SH	342	S53	850	S51	S52	
810	810	SII	S12	S13	820	S21	S22	S23	830	S31	832	833	OfS	SII	842	813	850	Sil	S52	853	S02	803	800	801	
S11	SII	S12	SI3	S10	S21	\$22	S23	820	831	\$32	833	830	SH	S42	843	07S	S51	S52	823	820	803	800	801	202	
S12	S12	SI3	810	S11	\$22	S23	S20	821	\$32	833	830	831	842	843	078	SH	S52	823	850	Sil	800	108	S02	803	
S13	S13	810	SII	813	S23	S20	S21	822	833	830	831	832	843	840	118	842	853	820	158	852	801	802	803	800	
820	820	S21	S22	S23	830	S31	S32	833	840	SIL	842	S43	820	Sil	852	S53	S02	803	008	108	212	813	810	S11	
821	S21	\$22	S23	820	831	832	833	830	S41	842	843	078	S51	852	823	850	803	800	108	208	SI3	810	S11	S12	
S22	S22	S23	\$20	S21	\$32	833	830	831	842	843	840	SH	852	853	850	S51	800	801	208	803	810	S11	S12	813	
S23	S23	S20	821	S22	833	830	831	832	843	018	S41	842	823	850	851	852	201	208	803	800	S11	S12	813	810	
830	830	831	S32	833	840	S41	\$42	S43	S50	S51	S52	S53	S02	S03	800	201	S12	813	810	SII	822	S23	820	S21	
831	831	S32	833	830	S41	S42	S43	07S	S51	S52	S53	S50	S03	800	201	S02	SIS	810	SII	212	\$23	\$20	S21	S22	
832	832	S33	S30	831	S42	S43	S40	S41	S52	S53	850	S51	800	201	S02	S03	810	SII	S12	813	\$20	S21	\$22	S23	
8333	833	S30	S31	S32	S43	S40	S41	S42	S53	850	S51	S52	S01	S02	S03	800	S11	S12	SI3	810	S21	S22	S23	S20	
840	07S	SII	S42	S43	S50	S51	S52	S53	S02	S03	800	S01	S12	813	810	S11	\$22	823	820	S21	S32	833	830	831	
S41	S41	S42	S43	S40	S51	S52	853	S50	803	800	801	S02	SI3	S10	811	S12	S23	820	821	822	833	830	831	S32	
842	S42	S43	S40	S41	S52	S53	S50	S51	800	201	802	S03	S10	SII	812	SI3	820	821	S22	823	830	831	832	833	
843	S43	S40	S41	S42	S53	820	S51	852	S01	802	803	800	S11	S12	SI3	810	S21	822	823	820	831	S32	833	830	
820	S50	S51	S52	S53	S02	S03	S 300	108	S12	SI3	810	S11	S22	\$23	830	821	833	833	830	831	S42	843	07S	841	
128	SSI	S52	S23	S50	S03	300	501	S05	S13	810	SII	S12	S23	\$20	S21	822	533	830	831	833	843	S40	841	842	
S52	S52	S53	\$ 850	S51	. S00	S01	S02	S03	S S10	S11	S12	SI3	820	S21	822	S23	830	S31	832	833	078	841	842	843	
823	S53	8 850	S21	S52	S01	S02	S S03	8 800	SII	S12	S13	S10	S21	822	S23	820	831	833	833	830	841	842	843	840	
	1													700		1					1000	S. Carlot			

These rules are also applicable to the manipulation of the associative and commutative operation. For the acquisition of an inverse element or segment, the following formulae are applied:

(1)
$$x = 0$$
: $(Sxy)^{-1} = S(12-x)(4-y)$
For example $(S01)^{-1} = S(12-0)(4-1) = S03$

(2)
$$x > 0$$
: $(Sxy)^{-1} = S(6-x)(6-y)$
For example $(S23)^{-1} = S(6-2)(6-3) = S43$

Now group equation is solvable as follows:

(1)
$$S23 \cdot x = S51$$
 $x = (S23)^{-1} \cdot S51 = S(6-2)(6-3) \cdot S51$
 $= S43 \cdot S51 = S(4+5-6)(3+1+2)$
 $= S32$
(2) $x \cdot S03 = S42$ $x = S42 \cdot (S03)^{-1}$
 $= S42 \cdot S(12-0)(4-3)$
 $= S42 \cdot S01$
 $= S43$

The multiplication table of this group (G_6^4) is shown in Table 9.

Figures 19 and 20 show diagram and structural models of a marriage alliance organized by eight patri-lines or hordes. They produced $4\times8=32$ segments. The number of the generation cycles of the matrilines is not four. It is determined by the *least common multiple* (*LCM*) of the generation cycle of patri-lines and the number of the patri-lines. Then the number of matri-lines is determined by the quotient of the total number of segments and the generation cycle of the matri-line. Thus in this case the matri-line is four and it is regulated by a 8-generation cycle. This principle is valid for application in any other case. This group (G_8^4) can be expressed as a set of 32 segments in the following diagram.

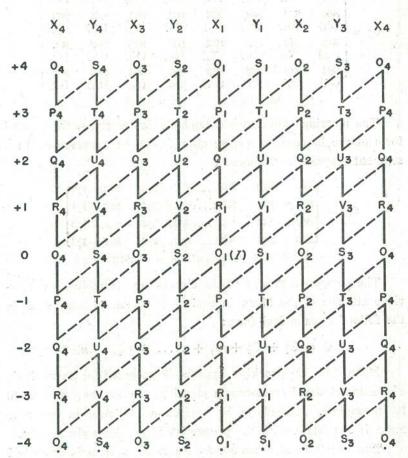


FIG. 19. Matrilateral cross-cousin marriage in Murngin system composed of eight hordes.

$$G_8^4 = \begin{cases} 800 & 810 & 820 & 830 & 840 & 850 & 860 & 870 \\ (O_1) & (T_2) & (Q_3) & (V_4) & (O_4) & (T_3) & (Q_2) & (V_1) \\ 801 & 811 & 821 & 831 & 841 & 851 & 861 & 871 \\ (P_1) & (U_2) & (R_3) & (S_4) & (P_4) & (U_3) & (R_2) & (S_1) \\ 802 & 812 & 822 & 832 & 842 & 852 & 862 & 872 \\ (Q_1) & (V_2) & (O_3) & (T_4) & (Q_4) & (V_3) & (O_2) & (T_1) \\ 803 & 813 & 823 & 833 & 843 & 853 & 863 & 873 \\ (R_1) & (S_2) & (P_3) & (U_4) & (R_4) & (S_8) & (P_2) & (U_1) \end{cases}$$

$$m: \mod = 4 \qquad f: \mod = 8$$

The marriage alliance is extendable limitlessly, so the general form of the Murngin marriage alliance can be represented in a segment diagram as follows:

$$G_{2n}^4 = \begin{cases} 800 & \text{S10} & \text{S20}, \dots, \text{S}(2_n-2)0 & \text{S}(2_n-1)0 \\ \text{S01} & \text{S11} & \text{S21}, \dots, \text{S}(2_n-2)1 & \text{S}(2_n-1)1 \\ \text{S02} & \text{S12} & \text{S22}, \dots, \text{S}(2_n-2)2 & \text{S}(2_n-1)2 \\ \text{S03} & \text{S13} & \text{S23}, \dots, \text{S}(2_n-2)3 & \text{S}(2_n-1)3 \end{cases}$$

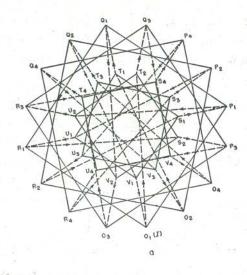
$$m: \mod = 4 \quad f: \mod = \text{LCM of } 4 \text{ and } 2n \quad n \geq 2$$

The compound groups of the circulation connubium or marriage alliance of the Murngin society (G) can be expressed in the following canonical form:

$$G = G_4^4 + G_6^4 + G_8^4 + \ldots + G_{2n}^4 \quad n \ge 2$$

Meanwhile, the working hypothesis proposed for the analysis of the matrilateral cross-cousin marriage system has proved to be incomplete. It needs to be revised as follows, in order to make it consistent with the properties of the Murngin system.

The matrilateral cross-cousin marriage system is described by an n-generation cycle or circulation, where the marriage alliance (or circulating connubium) is derived from n hordes or exogamous units, the minimum number for n being 3. When the generation cycle governing the hordes is fixed as p, and



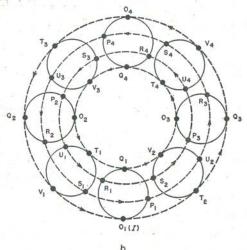


FIG. 20. Structural models for Murngin system composed of eight hordes.

 $p\geq 1$, then the other descent-line (if the former is patri-lineal, the latter is matri-lineal, or vice versa) will be regulated by a q-generation cycle, with q being the least common multiple of p and n.

This revised working hypothesis is also applicable for the analysis of other societies with matrilateral cross-cousin marriage systems. For example, in the Tarau society the patri-cycle p is 1, the number of clans organizing a marrige alliance n is 4, so the matri-cycle q is required as the least common multiple of p and p, that is, 4. Thus the kinship structure of the Tarau is expressed in the following group (G_4^1) :

 $G_4^1 = \{S00 \ S10 \ S20 \ S30\} \ m: mod=1, f: mod=4$

The structural model proposed by Dumont for the Murngin also fulfils the requirments of the hypothesis. In this case the

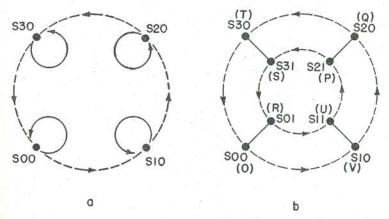


FIG. 21. Structural models of matrilateral cross-cousin marriage systems. a. Tarau b. Dumont's hypothetical Murngin system

patri-cycle p is 2, the number of patri-lines n is 4, the matricycle q is the least common multiple of p and n, that is, q is 4. This group (G_4^2) can be expressed as a set of 8 segments in the following diagram:

$$G_4^2 = \begin{cases} S00 & S10 & S20 & S30 \\ S01 & S11 & S21 & S31 \end{cases}$$
 $m: mod=2$ $f: mod=4$

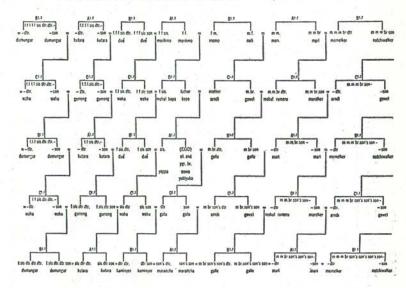
The Tarau are a living society, but the Murngin's structural model proposed by Dumont is fictional. For the realization of the latter, the society should strictly practice 'regular' and 'alternate' marriage in alternating generations. This would inevitably cause each horde to be divided into two patri-descont groups. Neither of these requirments fit into the Murngin society has been discussed in the previous chapters.

STRUCTURAL ANALYSIS OF KINSHIP TERMS

The table of the Murngin kinship terms by Warner (1930, 1937) is an extraordinary one, composed of five generations and seven lines of descent (see Table 10). Each horde is represented by a single patri-line, on which members of the same generation can be terminologically identified as two sex-distinguished units. But this elaborate kinship network does not complete a marriage alliance, both edges of the diagram still hang in mid-air. Not a few scholars have been involved in the problem of how to close the 'circulation connubium'.

According to the character of unilateral cross-cousin marriage the Murngin system requires many hordes to participate in the formation of a marriage alliance. From this point of view it is no wonder that Warner needs seven lines to depict the range of the Murngin kinship terminology. The Murngin marriage alliances, according to the discussion in the previous chapters, are organized by an unfixed number of hordes, but the number must be even and larger than 3. In the analysis of the Murngin system, Warner fails to see the problem of the formation of the marriage alliances. Therefore his Murngin model lacks a proper base. On the other hand, judged from another angle, Warner's table is in no way inappropriate, but it is limited only to the demonstration of the necessary and sufficient genealogical space to yield the whole set of the kinship terms. But it does not

TABLE 10 WARNER'S MURNGIN KIN TERM SYSTEM



show the full range of the Murngin kinship network or kinship space. Warner clearly mentioned that it is extendable limitlessly to include all tribal members. Warner has no intention of showing that the Murngin marriage alliance needs at least the patri-lines which he depicts in his diagram. This has already been discussed by Radcliffe-Brown (1951: 45–48) who has gone one step farther by proving that a kinship space with five matri-lines and five generations is sufficient for the demonstration of the complete set of kinship terms.

As mentioned above, the Murngin know of no bounds to limit their relatives. The network is extendable endlessly, until all of the tribal men are included. We must now extend our dis-

TABLE 11 LIST OF THE MURNGIN'S KIN TERMS

		M LDD	ELMIN	
arndi	arndi	ngandi	nandi	1
BAPA	bapa	bapa	1	baba
DUE/due	dué	duei	duwei	duwei
DUE-ELKER/due-elker	dumungur	dumungur	dumungur	dumungur
GALLE/galle	galle	galei	galei	galei
GATU/gatu	gatu	katu	. [gadu .
GAWEL	gawel	ganal	gaual or galwal	galwal
GURRONG/gurrong	gurrong	gorong	goron or gurun	gurung
KAMINYER/kamiyer	kaminyer	kaminyar	gominiar	gominiar
KUTARA/kutara	kutara	kutara	kutara or gudara	gudara
MARAITCHA/maraitcha	maraitcha	maraitcha	1	maraidia
MARI/mari	mari	mari	mari	mari
MARI-ELKER/mari-elker	marelker	maralkur	maralgur	maralgur
MARIKMO/marikmo	marikmo	mari'mor	marikmo	marimo
mokul bapa	mokul bapa	mukul bapa	1	1
mokul rumeru	mokul rumeru	mukul rumaru	mugul rumarun	1
momo	momo	mormor	1	1
momo-elker	momelker	mormalkur	mumulgur	1
NATI	nati	ngati	nadi	ngadi
NATI-ELKER	natchiwalker	ngatiwalkur	nadiwalgur	ngadiwalgur
WAKU/waku	waku	waku	wagu	wagu
WAWA	wawa	wawa		1
уерра	yeppa	yapa	1	1
Y UKIY UKO/yukiyuko	yukiyuko	yukuyuku	1	1

cussion to the terminology for the members beyond the scope of the seven patri-lines and five generations.

The Murngin use 24 kinship terms. But owing to the orthographical divergences caused by the dialects or reporters' preferences, it is quite hard in some cases to recognize their identity. Table 11 is a comparative list showing these differences. The terms on the left column are those adopted in this paper, with the male terms in capital letters and female in small letters; those common to both sexes are distinguished by initial capital letter. This spelling is applied to the first column only, and small letters are used for the other columns disregarding sex. According to the linguistic structure, kinship terms are distinguished as elementary, derivative, and descriptive (Murdock 1949: 98). Most of the Murngin kinship terms belong to the category of elementary terms. Of the others, only five belong to that of the derivative and one to the descriptive category. A suffix '-elker' (-unker or -alker) means 'small' or 'smaller ones' (Warner 1930: 210). Combining this suffix with the elementary terms, such derivative terms as NATI-ELKER, momo-elker, MARI-ELKER and Due-elker are produced; mokul is the only elementary term which is not used by itself, and in a narrow definition it means 'father's sister' or 'mother's mother's brother's daughter'. In case these two have to be distinguished, the former adds BAPA (means 'father') and a unique occurrence of a compound descriptive term results, whereas the latter adds rumeru, which means 'taboo'.

As the 24 kinship terms should be applicable to all relatives as far as the kinship network reaches, the seven patri-lines and five generations table proposed by Warner is apparently not suffcient. Subsequently the range is enlarged by Webb to eight lines and nine generations and by Elkin (1953), who made some further slight additions. But this still does not satisfy the requirement. Though the scope depicted in Warner's table is limited, his early report explains how the aborigines ingeniously achieve the fulfilment of the marriage alliance for which the table is defective. He writes:

"This asymmetrical cross-cousin marriage causes a male relative in the third patrilineal line to the left of ego and a female relative in the third patrilineal column to the right of ego to go unmated (in the kinship system) or a never ending addition to this system to bring about a symmetrical form, but as one line is added to each side of the system a new one is necessary, unless some device is created to throw this additional line back into the kinship. This has been done by the natives. Natchiwalker marries mari (not mari as mother's mother); gawel marries a distant mokul and to the left of ego dumunger marries another kutara, and waku another gurrong." (Warner 1930: 210-211)

However, Warner's description is explainable by Figure 22.

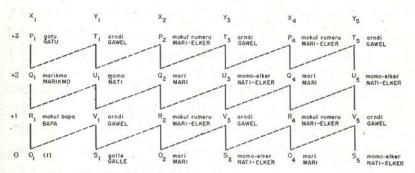


FIG. 22. Kinship terms extended to additional patri-lines

The NATI-ELKER (U₃) of line Y₃ ought to marry mari, but she should not become the mari of Q₂ of line X₂ which has been defined, but she should become the mari of another new line, the Q4 of X4 as shown in the Figure. In the same way, the GAWEL (V3) of line Y3 should marry mokul rumeru, who may not belong to the R2 of line X2 but must belong to the R4 of line X4. On the contrary, we may presume that NATI-ELKER of line Y₃ marries mari who belongs to line X2. Then, according to Warner's table the MARI of line X2 marries line Y3's momoelker. Thus sister-exchange marriage is taking place between lines X₂ and Y₃. If this kind of marriage is repeated generation after generation, then a bilateral cross-cousin marriage system will be established. But this will conflict with the fact that the Murngin system is regulated by the principle of matrilateral cross-cousin marriage and also causes the disorganization of the marriage alliance. The marriage of NATI-ELKER of line Y3 to mari of line X2 is not permissible. Meanwhile, Warner's table shows that MARI (X2) marries momo-elker (Y3), which allows us to presume that the spouse of MARI (Q4) of the new line X4 must be called momo-elker. Then, based on the above-mentioned analysis, this momo-elker does not belong to U3 of line Y2 but should belong to another new line, say, U5 of line Y5 on the Figure. (Strictly speaking, it is correct to say that line Y5 is not identical with line Y3, but judging from the structure of marriage alliance, there exists the possibility that line Y5 may be identified with one of the other patri-lines Y4 or Y2. We continue our discussion with the premise that the new line can not be identified with any line already existing.) For the same reason, the MARI-ELKER (R4) of line X5 ought to marry arndi who belongs to the V5 of line Y5. Part of this presumption has already been proved by Elkin (1953: 413) to be the case.

Thus we may tabulate the principles regulating the term system for the new patri-lines added to the right edge of Warner's table as follows:

- (1) The kinship terms of the newly added patri-lines on the right side which belong to the X moiety are identical with those of line X₂.
- (2) In the same way, those belonging to the Y moiety, are identical with that of line Y₃.

By the same procedure, the following principles are produced which are applicable to the left side of Warner's table. They are:

- (3) The kinship terms of the newly added patri-lines on the left side which belong to X moiety, are identical with those of line X₂.
- (4) In the same way, those belonging to the Y moiety, are identical with that of line Y₄.

The above-mentioned principles are expressed by the following formulae:

- (1) $X_2 = X_4, X_6, X_8, X_{10}, X_{12}, \ldots = X_{2n}$
- $(2) Y_3 = Y_5, Y_7, Y_9, Y_{11}, Y_{13}, \ldots = Y_{2n+1}$
- (3) $X_3 = X_5, X_7, X_9, X_{11}, X_{13}, \dots = X_{2n+1}$
- $(4) \quad \mathbf{Y}_{4} = \mathbf{Y}_{6}, \, \mathbf{Y}_{8}, \, \mathbf{Y}_{10}, \, \mathbf{Y}_{12}, \, \mathbf{Y}_{14}, \dots = \mathbf{Y}_{2n+2}$

Adding Webb's material, we expand the range of the kinship terms to a genealogical space composed of n patrilines with nine generations. In the following, let us proceed one step forward, to determine the terms for the genealogical space endlessly extending to ascending and descending generations. In actual life, a genealogical space with the depth of three to five generations covers the terms in use. But from the theoretical point of view, every possibility must be discussed, though the

search for the maximal range of each kinship term may to be redundant.

The patri-line X₁, to which Ego is subordinated, is equivalent to Radcliffe-Brown's '0 line' or Lawrence and Murdock's 'patri-line 4'. Following Webb's information, kinship terms applied to the nine generations are tabulated together for the relevant segments as follows:

TABLE 12a

PATRI-LINE X₁ (O or PATRI-LINE 4): TERMS
FOR SEGMENTS AND GENERATIONS

Segment	Kinship term		Generation	
O ₁	WAWA/yeppa	İ	0(>Ego)	+4
	Yukiyuko	-4	0(< Ego)	
$\mathbf{P_i}$	Gatu	-1		+3
\mathbf{Q}_1	Marikmo			+2
	Maraitcha	-2		
\mathbf{R}_{1}	BAPA/mokul bapa	3		+1

Concerning the terms of segment O₁, Warner gives different designations. He states that in Ego's generation, male should be distinguished by relative age, that is, WAWA for elder brother and YUKIYUKO for younger brother. On the contrary, no distinction is made for female, who are called yeppa. Webb revises Warner's designations by saying that those who older than Ego should be distinguished by sex and for those younger than Ego the sex should be ignored.

From this table, we see that the following three criteria are adopted by the Murngin. They are: (a) the criterion of sex, (b) the criterion of relative age, (c) the criterion of identity (the same term is applied to the members of the same segment, while the generation is ignored).

If the third criterion is applicable to any other segment beyond Webb's scope, then we can obtain the kinship terms for any generation in any patri-line. For example, if we apply the criterion of identity to the fifth ascending generation of patri-line X_1 belonging to segment R_1 , we may presume that $BAPA/mokul\ bapa$ are applicable to them.

The corelation of the segments, kinship terms and generations of the other patri-lines is shown in the following tables:

TABLE 12b
PATRI-LINE Y₁ (R-1, PATRI-LINE 3)

Segment	Kinship term	G	enerati	on
S_1	Galle	-4	0	+4
\mathbf{T}_{t}	GAWEL/arndi	-1		+3
$\mathbf{U_1}$	NATI/momo			+2*
	Galle	-2	- 10 17	g Trail is
\mathbf{v}_{i}	GAWEL/arndi	-3		+1

^{*} non-periodic

TABLE 12c
PATRI-LINE Y₂ (L-1, PATRI-LINE 5)

Segment	Kinship term	Ge	enerati	on
S_2	Due	-4	0	+4
\mathbf{T}_2	Waku	-1		+3
\mathbf{U}_2	Kaminyer	-2*		
	Due			+2
\mathbf{V}_2	Waku	-3		+1

TABLE 12d
PATRI-LINE X₂ (R-2, PATRI-LINE 2)

Segment	Kinship term	G	enerati	on
O ₂	Mari	-4	0	+4
Pa	MARI-ELKER/ mokul rumeru	-1		+3
\mathbf{Q}_2	Mari	-2		+2
\mathbf{R}_2	MARI-ELKER/ mokul rumeru	-3		+1

TABLE 12e
PATRI-LINE X₃ (L-2, PATRI-LINE 6)

,	Segment	Kinship term	G	eneratio	n
	O ₃	Kutara	-4	0	+4
	P_3	Gurrong	-1		+3
	\mathbf{Q}_3	Kutara	-2		+2
	\mathbf{R}_{3}	Gurrong	-3		+1

TABLE 12f
PATRI-LINE Y₃ (R-3, PATRI-LINE 1)

Segment	Kinship term	G	enerati	on
S_3	NATI-ELKER/ momo-elker	-4	0	-4
\mathbf{T}_3	GAWEL/arndi	-1		+3
$\mathbf{U_3}$	NATI-ELKER/ momo-elker	-2		+2
V_3	GAWEL/arndi	-3		+1

TABLE 12g
PATRI-LINE Y₄ (L-3, PATRI-LINE 7)

Segment	Kinship term	G	enerati	on
S_4	Due-elker	-4	0	+4
T ₄	Waku	-1		+3
\mathbf{U}_{4}	Due-elker	-2		+2
\mathbf{V}_4	Waku	-3		+1

Except the seven patri-lines shown above, the others are identifiable with the lines X_2 , X_3 , Y_3 and Y_4 , which are not listed again. Those terms marked by asterisks are non-periodic, that is, they are not applicable to those members who belong to the same segment but different generations.

Warner's seven patri-lines are simplified and transferred into Figure 23a. Each matri-line runs from the right upper to the left lower corner and intersects the vertical patri-lines at an angle of 45 degrees. If we take the intersecting point of the matri-line with the patri-line X_1 as a fulcrum and turn each

b

FIG. 23. Matrilateral cross-cousin marriage system. a. Generation is distinguished, regular form. b. Transformed: generation is ignored, matri-line is emphasized in horizontal position.

matri-line to become horizontal, forming a right angle with the vertical patri-lines. This shown in Figure 23b. Now each patri- and matri-line is extended to fill up the blanks and corresponding kinship terms are added to each segment. By this procedure Figure 24 is produced.

In this network-like diagram, column (solid line) stands for patriline and row (dotted line) for matri-line; Ego (I) is placed in the center (O_1) as origin or $zero\ point$, taking the vertical line which passes through the origin or Ego's patri-line X_1 as y-axis and the horizontal line passing through the origin or Ego's matri-line as x-axis; with the positive for ascending generation and negative for descending generation. Thus a two-dimentional Cartesian plain is constructed and each kinship term can now be demonstrated by a set of numbers. From this coordinate diagram the following simple equations are reduced:

WAWA/yeppa	Yukiyuko	
x = 0	x =	0
y = 4n	y =	-4n
BAPA/mokul bapa	Gatu	
x = 0	x =	0
$y = \pm 4n + 1$	y =	$\pm 4n + 3$
GAWEL/arndi	Waku	
x = 2n + 1	x =	-(2n+1)
$y = \pm 2n$	y =	$\pm 2n$
NATI/momo	Kaminyer	
x = 1	x =	-1
y = 1	y =	-1
Galle	Due	
x = 1	x =	-1

15	I mono-e	SGAWEL	FNATI-E	arndi 5 GAWEL	S NATI-E	gAWEL	F NATI-E	arndi 5 GAWEL	T NATI-E	S GAWEL	BOBO-6	ELKER
Ž	B mokul-r U	oth MARI	P. MARI-E	Q mari	.R, mokul-r U momo-e	mari MARI	P MARILE SNATILE	HARI	mokul-r	meri. v	mokul-r.S.	nelker ER, MAR
13	B momo-e	Tarndi	JATI-E	V arndi		Tz grndi	U BOHO-8 P	V arndi	S NATI-E	GAWEL	D NATI-E	lker, mon
Λ2	P mokul-r	Q mari	GALLE MARI-E	O2 meri	Packul-r.S. momo-e	Q mari	R2 MARI-E	O mari	P mokul-r	Q meri	R mokul-r u momo-e p	momo-e=momo-elker, momelker R MARI-E=MARI-ELKER, MARELKER
11		V arndi GAWEL	S galle	GAWEL	MORO	arndi GAWEL	S galle	gawel		arndi	S galle	omo
Δ1	R mokul-b u galle	O Yeppa	p gatu	Q marikmo T	Baba	yukiyuko O, yeppav (I) WAWA	gatu 1 GATU	MARAITCHA 1	R mokul-b galle BAPA GALLE	O yukiyuko v	gatu Sgalle	rumeru DUMUNGUR
03	:	weku Z WAKU	due 2 DUE	waku		1	KAMINYER KAMINYER	waku WAKU.	due	waku WAKU	due	kul ER,
241	gurrong s due	Rutara 7	R GURRONG U	,0 kutara	due-e P gurrong g due	kutara waku	R GURRONG U	o kutara v	gurrong S GURRONG	kutara T	gurrong U	DUE-E-DUE-ELKER,
	d due-e p	waku	due-e	waku	due-e	waku	due-e	waku Waku	due-e	V waku		
٠.	R gurrong D	kutara V	gurrong	kutara T	SULFORE U	kutara V	gurrong s	kutara T	R GULFONG U	kutara KUTARA	gurrong S due-e	
0	Se due-e	Te waku	due-e	Waku	due-e R &	waku o 5	due-e P	waku 6 waku	due-e	Te waku	due-e	due-e=due-elker, mokul-b=mokul
	e3	H	D.	Α.	en .	H	Þ.	P, -	eð.	H	- Þ.	due

FIG. 24. Kinship terms in Cartesian plane

$$y = \pm (2n+1)$$
 $y = \pm (2n+1)$ Exception: $y = 1$ Exception: $y = -1$

Marikmo

 $x = 0$ $y = 4n+2$ $y = -(4n+2)$

Mari

 $x = 2(n+1)$ $y = \pm 2n$ $y = \pm 2n$

MARI-ELKER/mokul rumeru

 $x = 2(n+1)$ $y = \pm (2n+1)$ $y = \pm (2n+1)$

NATI-ELKER/momo-elker

 $x = 2n+3$ $y = \pm (2n+1)$
 $y = \pm (2n+1)$
 $y = \pm (2n+1)$

Due-elker

 $x = -(2n+3)$ $y = \pm (2n+1)$

The order of terms follows their distance from Ego. Consequently, the two units of each row have the same absolute value. In other words, the right column is the inverse of the left column, and vice versa. They are standing in polar relationship. In the following, a term in the left column is called a 'positive term' and that in the right column a 'negative term'. Each has its 'opposite term' or 'inverse term' in the same row.

In the previous chapters, two basic units, father-child link m and mother-child link f have been proposed as *generators* for the analysis of the section system. They are also applicable to the analysis of the kinship terms. Supposing the number from descending to ascending generations as positive and the reverse as negative, the value of x is equivalent to the exponents of f, and the value of y to that of m. Terms of the same value are called 'pair' and terms of the same absolute value are called

'set'. Thus the Murngin's 24 terms can be classified into eighteen pairs and nine sets. They are expressed by the following formulae:

WAWA/yeppa $= m^{4n}$ Yukiyuko $= m^{-4n}$ BAPA/mokul bapa $= m^{\pm 4n+1}$ Gatu $= m^{\pm 4n+3}$

GAWEL/arndi = $m^{\pm 2n} f^{2n+1}$, $f^{2n+1} m^{\pm 2n}$

Waku = $m^{\pm 2n} f^{-(2n+1)}$, $f^{-(2n+1)} m^{\pm 2n}$

NATI/momo = mf, fm

Kaminyer $= m^{-1}f^{-1}, f^{-1}m^{-1}$

Galle = $m^{\pm(2n+1)}f$ (exception: mf),

 $fm^{\pm(2n+1)}$ (exception: fm),

Due = $m^{\pm(2n+1)}f^{-1}$ (exception: $m^{-1}f^{-1}$)

 $f^{-1}m^{\pm(2n+1)}$ (exception: $f^{-1}m^{-1}$)

Marikmo = m^{4n+2} Maraitcha = $m^{-(4n+2)}$

Mari = $m^{\pm 2n} f^{2(n+1)}$, $f^{2(n+1)} m^{\pm 2n}$

MARI-ELKER/mokul rumeru = $m^{\pm(2n+1)}f^{2(n+1)}$, $f^{2(n+1)}m^{\pm(2n+1)}$

Gurrong = $m^{\pm(2n+1)}f^{-2(n+1)}, f^{-2(n+1)}m^{\pm(2n+1)}$

NATI-ELKER/momo-elker = $m^{\pm (2n+1)} f^{2n+2}$, $f^{2n+2} m^{\pm (2n+1)}$

Due-elker $= m^{\pm (2n+1)} f^{-(2n+3)},$ $f^{-(2n+3)} m^{\pm (2n+1)}$

The genealogical space depicated in Figure 24 shows each kinship term to appear regularly, which enables us to apply mathematical devices to subordinate them to mathematical formulae. In the following, let us examine the structural model regulating the terminological system. We adopt the following symbols for the eighteen paires or nine sets.

MURNGIN

TABLE 13 KINSHIP TERMS AND SYMBOLS

Positive term	Symbol	Negative term Symbol
WAWA / yeppa	0	Yukiyuko
BAPA / mokul bapa	A	Gatu
GAWEL / arndi	n	Waku
NATI / momo	Δ	Kaminyer 🛕
Galle	\Diamond	Naminyer Due
Marikmo	ŏ	Maraitcha -
Mari	0	Kutara
MARI-ELKER / mokul ru	△	Gurrong
NATI-ELKER / momo-		Due-elker
		ффффф
***	A	·
· · · · · · · · · · · · · · · · · · ·		-фффф
		QQQQ
LAL	I I	
THE THE PERSON NAMED IN COLUMN 1	A 200	7 7 7 7
TIT		A A A A -

FIG. 25. Kin Term Symbols in Cartesian Plane

In Figure 25 we replace the terms of Figure 24 by these symbols. Based on this table the structure of each patri- and matri-line has to be discussed.

Among all patri-lines line X_1 has a unique and independent existance and does not share its terms with other lines. Furthermore, it is also the only line which strictly obeys the pattern of the 4-generation cycle. Starting from zero point, the upper and lower generations have special cycles. This is caused by the coexistence on the same line of relatives in reciprocal relationship. The terms for descending generations are the inverse terms of the ascending generations. These two cycles are shown in the following figures.

Upper:
$$\bigcirc \rightarrow \triangle \rightarrow \bigcirc \rightarrow \triangle \rightarrow \bigcirc$$

Lower: $\bigcirc \rightarrow \triangle \rightarrow \bigcirc$

The terms for patri-lines Y_1 and Y_2 belong to the same set, but are of positive and negative value respectively. For example, Y_1 in the upper right or the 1st quadrant and Y_2 in the 3rd quadrant; Y_1 in the 4th quadrant and Y_2 in the 2nd quadrant. On these two lines, there is a set of kinship terms NATI/momo and Kaminyer which appear once only. They are not periodic. If we presume that NATI/momo is Galle's and Kaminyer is Due's subtype respectively and if we transform the subtype into the original ones, then line Y_1 enters into the 2-generation cycle regulated by terms GAWEL/arndi and Galle alternately passing through the 1st and 4th quadrants; line Y_2 is now regulated by the inverse terms Waku and Due alternately passing throug the 2nd and 3rd quadrants. The term structures of lines Y_1 and Y_2 are shown in the following figures:



Patri-line X_2 and all the other patri-lines which belong to X_{2n} have an identical term structure, they are regulated by two pairs of positive terms, Mari and MARI-ELKER/mokul rumeru alternately passing through both the 1st and 4th quadrants. Those patri-lines belonging to X_{2n+1} have the inverse form of the former, regulated by Kutara and Gurrong alternately. Their cycles are expressed as follows:

$$X_{2n}$$
: $\bigcirc \longrightarrow \bigcirc \longrightarrow \bigcirc$ or $\bigcirc \longleftarrow \bigcirc$
 X_{2n+1} : $\bigcirc \longrightarrow \bigcirc \longrightarrow \bigcirc$ or $\bigcirc \longleftarrow \bigcirc$

Patri-lines belonging to Y_{2n+1} have the same structure which is regulated by NATI-ELKER/momo-elker and GAWEL/arndi alternately. Patri-lines Y_{2n+2} have the inverse form of the former, regulated by Due-elker and Waku alternately. They are:

$$Y_{2n+1}$$
: $\longrightarrow \bigvee \longrightarrow \bigcirc$ or $\longrightarrow \bigvee$ Y_{2n+2} : $\longrightarrow \bigvee \longrightarrow \bigcirc$ or $\longrightarrow \bigvee$

If we compare this type with that of Y_1 and Y_2 , we find that both of them belong to the alternate generations type. Each of the two sets has one pair of terms in common while the other is different. In other words, the Galle of Y_1 transfers to NATI-ELKER/momo-elker of Y_{2n+1} and Due of Y_2 transfers to Due-elker of Y_{2n+2} . In the previous line we presume NATI/momo to be a variant or subtype of Galle. As NATI-ELKER/momo-

elker are a derivative form of NATI/momo formed by adding the suffix elker, we may regard NATI-ELKER/momo-elker as a derivative type of NATI/momo which are variants or subtype of Galle. On the other hand, it goes without saying that Due-elker is the derivative form of Due formed by adding the suffix elker. If we conbine these derivatives, variants and proto-types, then any patri-line belonging to Y may be attributed to one of the patterns of term structure and fall into either the positive or the negative type.

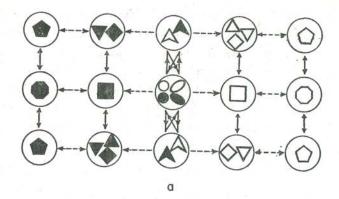


Concerning the application of the proto-type, variants and derivative forms in practical usage, the rules are:

/f/=1, the proto-type is used; /f/=/m/=1, then the variants should be chosen; /f/=2, the derivative forms are applied.

Our discussion permits us to limit the structural model of the Murngin term system to a finite 4-quadrant Cartesian plain as shown in Figure 26a. If the positive and negative value of the terms is ignored, and the patri-line is emphasized, a new structural model emerges (Figure 26b).

The system of the Murngin kinship terminology, endlessly extending on the infinite f m network, is really a kaleidoscope. But in the core, the system is organized by three patri-lines X_1 , Y_1 and X_2 only, belonging to the 1st quadrant. By the manipulations of repetition, mirror image and opposite or inverse terms the whole system is amplified. On the 1st quadrant, those



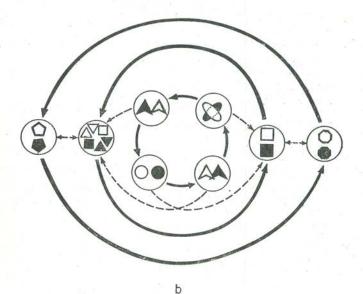


FIG. 26. Structural models of Murngin kin term system.

- a. Positive and negative term in different position.b. Positive and negative term in same position.

lines belonging to Y_{2n+1} are merely the repetition of the line Y₁, X_{2n} are the repetition of X₂; the 4th quadrant is the mirror image of that of the 1st quadrant; the 3rd is composed of the opposite terms of the 1st quadrant; the 2nd by those of the 4th quadrant. Among the three basic patri-lines, X₁ is composed of unique terms and two 4-generation cycles, the one for descending generations being the inverted image with opposite terms to that of the ascending generations. In this case the 4-generation cycle of terms coincides with that of the subsection system, the former apparently the product under the impact of the latter. The other lines, all follow the 2-generation cycle by adopting 2 pairs of terms for alternating generations. The inner circle is regulated by the complicated 4-generation cycle and the other two by the simplified 2-generation cycles. This phenomenon may be a natural occurence, or it may have a particular significance still to be explained. For example, originally the 2-generation cycle might have been the ruling principle, but later it suffered from the impact of the 8-section system, and structural change have taken place in the central part. It is also may have been caused by the opposite reason, being originally ruled by the 4-generation cycles, but by applying the same term to the two sections of opposite moiety which were allowed to marry the principle was caused to change.

Among the eighteen pairs of terms, with some of them sex is distinguished and with others sex is ignored. There are some criteria to be obeyed. They are listed as follows:

- (1) All of the negative forms ignore sex.
- (2) Positive term are determined by their primary designation:

- For the terms designating one generation ascending fron Ego sex is distinguished.
- b) For the terms designating two generations ascending from Ego sex is ignored.
- For the terms designating the descending generations sex is ignored.
- (3) Terms assigned to Ego's generation:
 - a) For those who are older than Ego and belong to Ego's line, sex is distinguished.
 - b) Under the same condition, for those who are younger than Ego sex is ignored.
- c) For those who do not belong to Ego's line sex is ignored. Based on these criteria, a positive term Mari which designated the two ascending generations needs not distinguish sex in principal. But for the term one generation descending from the Mari sex has to be distinguished. So the term MARI-ELKER, which is derived from the Mari is applicable to the masculine only, and for the feminine mokul is adopted. Thus a pair of terms MARI-ELKER/mokul rumeru is produced. Of all the positive terms two pairs do not fit the above-mentioned criteria. They are NATI/momo and NATI-ELKER/momo-elker standing for the two ascending generations. From the theoretical point of view if lines Y2n adopt Galle-elker (if a new term is allowed to be produced) which is the derivative of Galle, instead of NATI-ELKER/momo-elker, which is derived from NATI/momo, it fits the system correctly. But, why is the Galle-elker not employed instead of the NATI-ELKER/momo-elker?

MARRIAGE ALLIANCE AND KINSHIP TERMS

Above we discussed the two major parts of the Murngin system, the mathematical structures of marriage alliance and kinship terminology. Both systems seem to have contradictory properties. The characteristic of the former is that it is a 'closed' system, which needs a definite number of hordes to accomplish the cycle of marriage, and the latter is an 'open' system, wherein the network is extendable endlessly. Since both systems are coexistent within the same kinship structure, they should not exclude but on the contrary complement each other. However, to understand this mechanism, the settlement of the question of how the term network closes its edges under the frame of the marriage alliance is a prerequisite. This is the major subject to be discussed in this chapter.

Based on the presumption we made in previous chapters, the Murngin system should be organized by multiple marriage alliances, which are not isolate but intersect each other. Therefore people are enabled to extend the kinship network to cover the whole tribal membership through marriage alliances. Here, for the convenience of discussion a marriage alliance formed by 2n patri-lines or hordes, G_{2n}^4 (Figure 27) is designed to test this problem. In the Figure, X_1 is the unit where Ego is placed; the right (Y_1) is X_1 's wife-giver, and the left (Y_2) is X_1 's wife-taker. We refer to those on the right-hand as 'giver's side' and left-hand

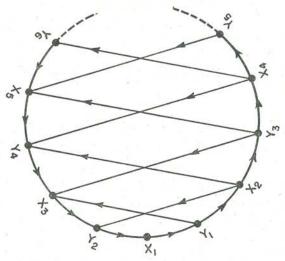


FIG. 27. Circulating connubium.

as 'taker's side'. If we superpose giver's side's X_2 on taker's side's X_3 to form one unit and remove the remainder, then the smallest marriage alliance G_4^4 consisting of four units X_1 , Y_1 , Y_2 and X_2 (or X_3) is produced. Similarly, by superposing giver's side's Y_3 on taker's side's Y_4 and removing the remainder, a marriage alliance G_6^4 is produced. In the same way, by superposing X_4 on X_5 a marriage alliance G_8^4 is produced. Following this procedure the other kinds of marriage alliances are producible.

Among the nine sets and eighteen pairs of kinship terms, only one set, NATI-ELKER/momo-elker and Due-elker is excluded from a marriage G_4^4 . This is the only exceptional case. In the other cases all of the nine sets of terms are used. However, the following predicament occurs when determining the terms to be applied to the last unit which is produced by the method of

superposition as mentioned above. That is, to which terms preference should be given, either to the giver's side's positive term or to the taker's side's negative term? These superposed units have the same kinship distance from X₁, so they are composed as a set of positive and negative terms. Suppose both terms are applicable to the superposed unit, then such a phenomenon as addressing each other by the same term arises. In the marriage alliance G_{2n}, Ego addresses X₄ of the same generation as Mari, and X5 as Kutara, reciprocally X4 addresses Ego as Kutara and X5 addresses Ego as Mari. Now X4 and X5 are superposed to form a marriage alliance G4, and Ego addresses the superposed unit by both Mari and Kutara, then in return they also address Ego by the same set of terms, Mari and Kutara. This phenomenon severely violates the criterion of polarity, according to which a set of positive and negative terms should be used respectively. This predicament could be avoided by the observation of some criteria provided by the aboriginal society. In the past, owing to the negligence of fieldworkers with regard to marriage alliance, no correct answer has ever been reported. This blank must be filled by theoretical assumption.

Let us re-examine the method of superposition again. Though this method fits the manipulation of a model, it is not suitable for the representation of an actual case. What is really happening is that a marriage relationship is established between the opposite moieties of giver's side and taker's side. For example, if a marriage takes place between X_2 and Y_2 , the former becomes the latter's wife-taker, then the four units X_1 , Y_1 , Y_2 and X_2 organize the smallest marriage alliance G_4^4 . In this case the 'marriage distance' between X_1 and X_2 on both sides is equal, so

 X_1 is assignable to the taker's side and X_2 the giver's side or vice versa, either is possible. But X_2 belong to the giver's side of X_1 in the original marriage alliance G_{2n}^4 , so it is ideal to adopt the same role on 'giver's side' in this new marriage alliance G_4^4 to avoid the confusion raised by similar marriage distance. In the same way, a marriage takes place between X_3 and Y_1 , the former becomes the latter's wife-giver, then the four units X_1 , Y_1 , Y_2 and X_3 form another marriage alliance G_4^4 . At this occasion X_3 should be addressed by negative terms to play the same role as in the original marriage alliance G_{2n}^4 .

For convenience of discussion, X1 where Ego is placed is called the 'original pole' a new unit which is produced by the superposition of X2 and X3 is called the 'oppsite pole'. Between the two kinship distance is equal on both side. If a positive term is applied to the opposite pole, we speak of a 'positive system'; on the contrary if a negative term is applied, we speak of a 'negative system'. Following this procedure, we may determine the system of the other marriage alliances. For example, X₃ becomes Y₃'s wife-giver, or Y₄ becomes X₂'s wife-giver, and marriage alliance G₆ are produced. The former's opposite pole is Y3, adopting a positive system (Y3 belongs to giver's side); the latter's opposite pole is Y_4 (taker's side), in a negative system. The formation of marriage alliance G_8^4 , G_{10}^4 and so on and their terminological conbinations are presumable in the same way. Lawrence and Murdock (1949), based on Webb's data, proposed a theory that eight patri-lines should accomplish a marriage alliance, where the eighth line (equivalent to the opposite pole in this case) adopts a negative system. Elkin's critique (1953) is based on the terminological network given to marriage alliance

 G_{2n}^4 . Both Webb's and Elkin's field work seem only to have touched one part of the Murngin system, and do not cover the whole dimensions.

The above are cases to show that the difference of the kinship distance between the 'giver's side' and 'taker's side' is 1. If the difference of kinship distance is 2 or greater than 2, another predicament is raised. Suppose Y₁ becomes X₅'s wife-taker and X_1 , Y_1 , X_3 , Y_4 and X_5 compose a marriage alliance G_6^4 . In this case Y4 becomes the 'opposite pole' of this circle while X5 which originally belongs to the 'taker's side' is now inevitably moved to the 'giver's side' so that the kinship distance of both sides may be balanced. In other words, X5 in the original circle is addressed as Kutara, but in the new circle must be called Mari. Thus X₅ is apparently addressable by both positive and negative terms. This causes another confusion of the terminological system. One of the methods of avoiding this disorder is to ignore one of the two marriage alliances, either the new one or the old one. The decision of which to drop may follow convenience and individual circumstances.

Based on this discussion, we conclude that the opposite pole is addressable either by positive or negative terms. The combination of terms and segments of each marriage alliance is tabulated as follows:

TABLE 14

							000	TWO ASSESSMENT AND MEANING		060	Land Tilly
	S00 (WAWA/yeppa, Yukiyuko)	ppa, Yukiyu	ko)	S10 (Waku)	Waku		820	S20 (Kutara or Mari)		200	S30 (GAWEL/arndi)
	S01 (Gatu)	•	(55)	SII (I	Kamin	(Kaminyer, Due)	S21	(Gurrong or MARI-E/mokul-r)	RI-E/mokul-	r) S31	(Galle)
II 5*	S02 (Marikmo,	(Marikmo, Maraitcha)	(ATT)	S12 (V	(Waku)		S22	(Kutara or Mari)		S32	(GAWEL/arndi)
	S03 (BAPA/mokul-b)	kul-b)		S13 (I	(Dne)		SS	(Gurrong or MARI-E/mokul-r)	RI-E/mokul-	-r.) S33	(NATI/momo, Galle)
_	S00 (WAWA/yeppa, Yukiyuko)	Yukiyuko)	S10 (Waku)		80°C	S20 (Kutara)	S30 (Waku o	S30 (Waku or GAWEL/arndi)	S40 (Mari)		S50 (GAWEL/arndi)
	S01 (Gatu)		S11 (Kaminyer, Due)	yer, Du		S21 (Gurrong)	S31 (Due-e o)	S31 (Due-e or NATI-E/momo-e) (MARI-E/mokul-r)	S41 (MARI-E,	/mokul-r)	S51 (NATI/momo, Galle)
 5°	S02 (Marikmo, Maraitcha)		S12 (Waku)		800	S22 (Kutara)	S32 (Waku o	S32 (Waku or GAWEL/arndi)	S42 (Mari)		S52 (GAWEL/arndi)
-	S03 (BAPA/mokul-b)		SI3 (Due)		w =	(Gurrong)	S33 (Due-e o	S33 (Duc-e or NATI-E/momo-e) (MARI-E/mokul-r)	S43 (MARI-E	/mokul-r)	S53 (Galle)
	S00 (WAWA/veppa Yukiyuko	S10 (Waku)		S20 (Kutara)	ra)	S30 (Waku)		(Kutara or Mari)	S56 (GAWEL) arndi	S60 (Mari)	S70 (GAWEL/arndi)
	S01 (Gatu)	S11 (Kaminyer, Due)	, Due)	S21 (Gurrong)	(Suo	(Due-e)	Gurr (Gurr	Gurrong or MARI-E	S51 (NATI-E) momo-e)	S61 (MARI-E) (mokul-r)	S71 (Galle)
ii 5°°	S02 (Marikmo (Maraitcha)	S12 (Waku)		S22 (Kutara)	ra)	S32 (Waku)		S42 (Kutara or Mari)	S52 (GAWEL) arndi	S62 (Mari)	S72 (GAWEL/arndi)
	S03 S13 (BAPA/mokul-b) (Due)	SI3 (Due)		S23 (Gurrong)	(Buo	S33 (Due-e)	S43 (Gurr	S43 (Gurrong or MARI-E) mokul-r	S53 (NATI-E) (momo-e)	S63 (MARI-E) (momo-e)	ST3 (NATI/momo, Galle)

In the general form of marriage alliance G_{2n}^4 , the opposite pole is expressible as $(n0 \ n1 \ n2 \ n3)$, where n is 2 or greater than 2. If n is an even number, the opposite pole adopts those terms $Mari-MARI-ELKER/mokul\ rumeru$ for the positive system and Kutara-Gurrong for the negative system. On the contrary, if n is an odd number NATI-ELKER/momo-elker-GAWEL/arndi are adopted for the positive and Due-elker-Waku for the negative system.

The Murngin kinship network is extendable to include all of the hordes, but they are reduced to two moieties, *Due* (X) and *Yiritcha* (Y). However many hordes may join in a marriage alliance, the terms applying to 'Ego's moiety' and the 'opposite moiety' are apparently different as shown in the above-listed tables. Thus Warner states that in the Murngin's mind, people are divided into different categories according to the moieties (1937:31). The distribution of terms in Ego's and the opposite moiety is shown as follows:

TABLE 15.
MOIETY AND KINSHIP TERMS

Ego's mo	iety	Opposite moiety		
Positive term	Negative term	Positive term	Negative term	
WAWA/yeppa BAPA/mokul bapa Marikmo Mari	Yukiyuko Gatu Maraitcha Kutara	NATI/momo GAWEL/arndi Galle NATI-ELKER/ momo-elker	Kaminyer Waku Due Due-elker	
MARI-ELKER/ mokul rumeru	Gurrong			

Among the five sets of terms belonging to Ego's moiety, the first three sets are monopolized for the use of members of Ego's hordes, and for the other hordes the last two sets of terms are applied. In both cases for the giver's side the positive term is applied and for the taker's side the negative term is applied. The four sets of terms applied to the opposite moiety, as mentioned in the previous chapter, are reducible to two sets of terms. That is, in the positive system, NATI-ELKER/momo-elker is a derivative type of NATI/momo and NATI/momo is presumably a variant or subtype of Galle; for the negative system, Due-elker is a derivative type of Due and Kaminyer is deducible as a variant of Due. Thus they are reduced to the two sets, Galle—Due and GAWEL/arndi—Waku. In the following, we adopt segment symbols and numerical notation system to study the relationship between terms and sections.

In Figure 26b we present a structural model of kinship terms adopting both positive and negative terms in the same position. This model is composed of three circles. The inner one is the cardinal class, which is represented by E, composed by the four segments of Ego's patri-line and regulated by a 4-generation cycle. The middle circle is composed of all of the parti-lines belonging to the opposite moiety, divided into two classes which are symbolized by B and C separately. and regulated by a 2-generation cycle. The outside circle is bisected into classes A and D, where they are composed of all patri-lines belonging to Ego's moiety except the line in which Ego is placed; the 2-generation cycle is also its ruling principle. Each circle is regulated by generator m and the relation of circle to circle by generator f. The distribution of segments among the five classes A, B, C, D and E is shown by the cases G_4^4 , G_6^4 , G_8^4 and G_{2n}^4 in the following tables.

TABLE 16. TERM STRUCTURE REPRESENTED BY NUMERICAL NOTATION SYSTEM

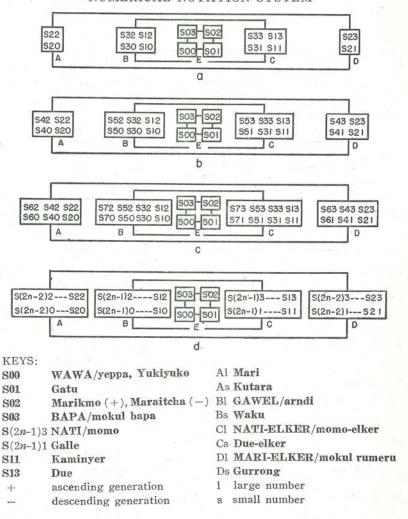
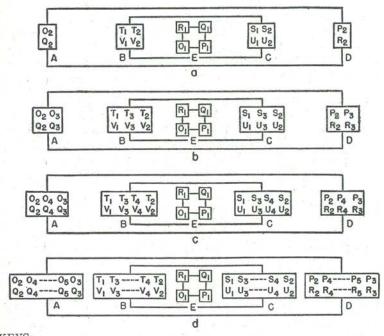


TABLE 17.
TERM STRUCTURE REPRESENTED BY SEGMENT SYMBOLS



KEYS:

- O1 WAWA/yeppa, Yukiyuko
- P1 Gatu
- Q₁ Marikmo (+), Maraitcha (-)
- R, BAPA/mokul bapa
- S₁ Galle
- S₂ Due
- U1 NATI/momo
- U2 Kaminyer

- O Mari, Kutara
- P MARI-ELKER/mokul rumeru, Gurrong
- Q Mari, Kutara
- R MARI-ELKER/mokul rumeru, Gurrong
- S NATI-ELKER/momo-elker, Due-elker
- T GAWEL/arndi, Waku
- U NATI-ELKER/momo-elker, Due-elker
- V GAWEL/arndi, Waku

As shown in each table, the cardinal class is constant and never affected by the increase of the number of patri-lines participating in the marriage alliance. E is composed of the four elements or segments with initial zero, as

$\mathbf{E} = \{ \mathbf{S00} \ \mathbf{S01} \ \mathbf{S02} \ \mathbf{S03} \} \text{ or } \{ \mathbf{O}_1 \ \mathbf{P}_1 \ \mathbf{Q}_1 \ \mathbf{R}_1 \}$

This class satisfies the following properties:

- (1) A new element which is produced by the binary operation of any two elements belonging to this class also belong to it.
- (2) Each element has its reverse in the set.

According to group theory, if a partial assemblage within a group can satisfy the above two properties, it is called a 'subgroup'. Thus that the class E is a subgroup of group G_{2n}^4 is very clear. But E is different from the other two subgroups $\{G_{2n}^4\}$ and $\{S00\}$, so it is specially called a 'true subgroup' of group G_{2n}^4 . Furthermore, each element has its inverse in the subgroup. If we call the conbination of each element with its inverse a 'couple', the four elements of the given subgroup compose three couples, corresponding to the three positive-negative term sets. For each couple, the element belonging to the ascending generation or senior of the same generation is assigned as a 'regular unit' and its inverse as a 'inverse unit'. The regular-inverse combination of three couples of class E is shown as follows:

TABLE 18.
COUPLES AND TERMS IN CLASS E

Regular unit	Positive term	Inverse unit	Negative term
S00 (O ₁)	WAWA/yeppa	S00 (O ₁)	Yukiyuko
S03 (R ₁)	BAPA/mokul bapa	S01 (P ₁)	Gatu
S02 (Q ₁)	Marikmo	S02 (Q ₁)	Maraitcha

Except for the four segments shown above, the others are assortable into four classes based on the combination of numerical numbers as even-even, even-odd, odd-even and odd-odd. They correspond to the four classes of A, B, C and D.

A is an assemblage of even-even numbers where mother's mother is subordinated. If section symbol Q represents the assemblage of all segments Q_1 , Q_2 , Q_3 and so on, A is expressible in the following diagram:

$$A = \begin{cases} S(2n-2)2 & S(2n-4)2 & S(2n-6)2 \dots S22 \\ S(2n-2)0 & S(2n-4)0 & S(2n-6)0 \dots S20 \end{cases} \text{ or } \{P \ Q\} \ n \ge 2$$
Exception: S02 and S00, or P₁ and Q₁.

Since this assemblage is not a closure system, it can not be a subgroup. But, each segment comprises its inverse, and if we assign the giver's side as 'regular' and the taker's side as 'inverse', then all of the regulars are called *Mari—MARI-ELKER/mokul rumeru*, and the inverses *Kutara—Gurrong*.

B is an assemblage of odd-even numbers to which mother belongs. They are:

$$\mathbf{B} \ = \ \begin{cases} \mathbf{S}(2n-1)2 \ \mathbf{S}(2n-3)2 \ \mathbf{S}(2n-5)2...\mathbf{S}12 \\ \mathbf{S}(2n-1)0 \ \mathbf{S}(2n-3)0 \ \mathbf{S}(2n-5)0...\mathbf{S}10 \end{cases} \text{ or } \{\mathbf{T} \ \mathbf{V}\}$$

This assemblage is also not a closure and only satisfies the axiom of inverses. The regulars are called *GAWEL/arndi* and the inverses *Waku*.

C is an assemblage of odd-odd numbers to which spouse (wife) belongs. This assemblage is composed of three sets of terms. They are:

TABLE 19.						
COUPLES	AND	TERMS	IN	CLASS	C	

Regular unit	Positive term	Inverse unit	Negative term
S13-1* (S ₁)	Galle NATI/momo NATI-ELKER/ momo-elker	S13 (S ₂)	Due
S11-1** (U ₁)		S11 (U ₂)	Kaminyer
Others		Others	Due-elker

^{*} In case n is an even number, the formula is: S(2n-1)1, if n is an odd number: S(2n-1)3.

Among the three sets of terms there exist proto, variant and subtypes as mentioned above.

D is an assemblage of even-odd numbers. It can be expressed in the following generalized form:

$$D = \begin{cases} S(2n-2)3 & S(2n-4)3 & S(2n-6)3...S23 \\ S(2n-2)1 & S(2n-4)1 & S(2n-6)1...S21 \end{cases} \text{ or } \{P \ R\}$$

Only one set of terms is used for this group. The regular (giver's side) is MARI-ELKER/mokul rumeru and the inverse (taker's side) Gurrong.

Generalizing the above analysis, the Murngin's kinship terms are classifiable into two major categories. The first one which includes some special segments mainly concentrates on the three patri-lines of X_1 , Y_1 and Y_2 , whereas parts of the last two and all of the other lines form the second categories.

Those segments belonging to the first category are: sibling (O_1) , father (R_1) , child (P_1) , father's father or son's child (Q_1) , wife (S_1) , husband (S_2) , mother's father or father's mother (U_1) , and sister's son's child or daughter's child (U_2) . These eight

^{**} Inverses with the upper case.

segments are composed of five sets of regular-inverse terms. The collation of sections and terms is shown as follows:

O ₁ >Ego	WAWA/yeppa	O ₁ <ego< th=""><th>Yukiyuko</th></ego<>	Yukiyuko
R_1	BAPA/mokul bapa	P_1	Gatu
$Q_1(+)$	Marikmo	$Q_1(-)$	Maraitcha
S_1	Galle	S ₂	Due
U ₁ (+1)	NATI/momo	U_1 (-2)	Kaminyer

But, U_1 (two generations descending) is also called *Galle* and U_2 (two generations ascending) *Due*, so the *Galle—Due* set, strictly speaking, does not qualify for classification in the first category. But this set appears only in patri-lines Y_1 and Y_2 , and never in other lines, so it also does not fit the second category. Based on their narrow senses, we temporarily classify them in the first category for convenience.

If the first category is assigned to the 'descriptive' system, then the second category belongs to the 'classificatory' system. Terms belonging to the last class are determinable not by segments but by sections. The collation of sections and terms is as follows:

TABLE 20.
COLLATION OF SECTIONS AND TERMS

Sec	tion	Number	Wife-giver	Wife-taker
0	Q	Even-Even	Mari	Kutara
P	R	Even-Odd	MARI-ELKER/mokul rumeru	Gurrong
S	U	Odd-Odd	NATI-ELKER/momo-elker	Due-elker
T	V	Odd-Even	GAWEL/arndi	Waku

The combination of sections and terms is not constant but relative, it is changed by the transition of Ego's place in the sections. The combination of numbers which are shown in the second column and terms, on the contrary, are invariable. They are interpreted as follows:

Even-Even: Those who belong to Ego's moiety, in Ego's gener-

ation or two generations ascending and descending.

Even-Odd: Those who belong to Ego's moiety, in one gener-

ation ascending or descending from Ego.

Odd-Odd: Those who belong to the other moiety, in Ego's

generation, or two generations ascending and

descending.

Odd-Even: Those who belong to the other moiety, in one

generation ascending or descending from Ego.

The kinship system of the Australian aborigines is well known for the capacity of comprehending the whole tribal membership into one organization. The Murngin section and terminological structure exactly fit this requirment. If two tribal men who never met before come across each other, the first step they should do is to inform each other of their section. And which kind of term (of course these belong to the second category) should be applied is determined at once. The second step is to discuss who belongs to the giver's side or to the taker's side; by tracing the hordes their relationship can be found. Then who should use positive or regular terms and who the inverse is determined.

KINSHIP PROBLEMS

In this paper based on the working hypothesis proposed for the matrilateral cross-cousin marriage system, the mathematical model of the Murngin system is designed and its properties are discussed. Is this model effective for the analysis of the marriage problem? In Warner's work many cases of the so-called 'wrong marriage' are reported. Some of them are thought to be insolvable (Barnes 1967: 35–36). Here one of the most notorious cases, in which is created a MARI—MARI-ELKER line in Ego's horde caused by wrong marriage, is chosen for the analysis. Warner writes:

"A clan that would for a period have only a mari and marelker, and a mari and mokul for the male and female relatives in one's clan other than one's own patrilineal line, could, by a wrong marriage of the male mari—let us say, to a waku—create a gurrong—Kutara line of descent for ego on the old mari—marelker clan." (1937: 27, n. 13)

Warner's description is too obscure and he does not add any further explanation to state why MARI marrying waku can create a MARI-MARI-ELKER line in Ego's horde, so this does not make it easy for readers to understand the actual circumstance. On the contrary, the fact that a new line is created in Ego's line incurs scholars' attention, and furthermore their doubt that Warner's seven line construction does not mean that seven

hordes are necessary for the formation of a marriage alliance, but that it shows kinship category only. Thus Leach (1961) proposes a hypothesis concerned with 'local group' and 'local line', to insist that one local goup must admit several local lines to exist. But the Murngin society does not allow two or more than two lines to coexist, Warner states this clearly following the above cited lines.

For this problem of wrong marriage, group equations are applicable for the solution. First we must clarify Warner's ambiguous description. That MARI marries waku is explainable in two ways: MARI marries his own waku, or MARI marries Ego's waku. For the former we may ask, "Who is MARP's waku?" or "Who is my MARPs waku to me?" For the latter, we may ask, "Who is my waku to my waku to my waku?" We now assume that this case occurs in a society organized by marriage alliance G_8 , and we apply the numerical notation system to manipulate the transition of kinship relationships. Warner on the previous page (1937: 26) mentioned that waku refers to mother's mother's brother, which is equivalent to numerical notation waku refers to sister's daughter which is equal to waku refers to sister's daughter which is equal to waku solvable by the following equation: waku the first question is solvable by the following equation: waku the answer is:

 $x = a \cdot b$ $= 860 \cdot 810$ = 870

S70 refers to *arndi*, that is the mother. This means Ego's father and mother practice wrong marriage. Mother is father's sister's daughter who practice uncle/niece marriage, Ego is the issue of

this couple. This wrong marriage really happens in Ego's horde. According to the Murngin's marriage rule, in case of a wrong marriage, the issues of this couple should be called according to their mother's relationships. Thus father's brother calls Ego sister's daughter's child, that is, *Kutara* (S20). Ego calls father's brother mother's mother's brother, that is, *MARI* (S60). Thus in the same patri-line, among the siblings if one practices a wrong marriage, those who practice regular ones form a *MARI*—*MARI-ELKER* line and wrong ones form a *Kutara—Gurrong* line.

As for the second question, equation $a \cdot x = b$, $x = a^{-1} \cdot b$ is available. They are:

$$S60 \cdot x = S10$$

 $x = (S60)^{-1} \cdot S10$
 $= S20 \cdot S10$
 $= S30$

Ego's waku (S10) is also MARI's waku (S30), and the generation difference expands to three; this kind of match is not so usual. In this match, the issue belongs to S40 (Kutara), thus this marriage can not happen in Ego's patri-line.

Recently Shapiro (1968: 346-353) reports the most extraordinary fact from Arnhem Land that a sister's daughter's daughter exchange marriage is practiced in the Murngin society. But to the readers of present paper this information needs no more be an enigma or proof that social change has taken place as Lévi-Strauss and Barnes believe. Shapiro clearly states that this marriage regulation happened in the marriage alliance organized by six hordes. However, Shapiro seems not to realise the special characteristics of this marriage alliance. As we have discussed

in the previous chapters, G_6^4 is a unique and quite extraordinary marriage alliance regulated by two matri-lines only, where in other cases the number of matri-lines is four or more than four. Thus in the case of G_6^4 members are divisible into two matrimoieties where exchange is inevitably practiced. Based on the clue offered by Shapiro, let us clarify the essential qualities this system involves.

As Figure 18 shows, matrilateral cross-cousin marriage is strictly obeyed. Then we may ask: "Is the sister's daughter's daughter exchange marriage a necessary concomitant secondary feature of this system?" First, the question arises: "Who is the husband of my sister's daughter's daughter?" In the numerical notation system husband is S13 (DUE) and sister's daughter's daughter is S20 (kutara). The equations is:

$$x = S20 \cdot S13$$
$$= S33$$

He is NATI-ELKER or DUE-ELKER. The second question is, "What is my NATI-ELKER's (or DUE-ELKER's) sister's daughter's daughter to me?" The answer is:

$$x = S35 \cdot S20$$
$$= S53$$

S53 is galle, that is, Ego's wife. This is also questionable as: "What is my wife to my NATI-ELKER?" The answer is:

$$S33 \cdot x = S53$$

 $x = (S33)^{-1} \cdot S53$
 $= S(6-3)(6-3) \cdot S35$
 $= S33 \cdot S53$

$$= S(3+5-6)(3+3+2)$$

= S20

Thus Ego's wife (S53) is NATI-ELKER's (S33) sister's daughter's daughter (S20) and NATI-ELKER's wife is Ego's sister's daughter's daughter (S20). This is not valid to any other marriage alliance except G_6^4 .

Shapiro also lists the kinship terms applied to matri-lines, but owing to the fact that the case he studied is not a perfect one, he feiled to find that the matri-line is regulated by a 12-generation cycle and the regular term orders.

In this paper we have approached the Murngin problem by a new mathematical method and structural models. If this approach is acceptable, the first hypothesis on which this paper is based has proved its workability. Though the other two hypotheses are not discussed here, and they are in need of some clarification, their reliability seems beyond doubt. Through this approach the apparent contradictions of the Murngin case can be seen to be, on the contrary, perfectly regular and controllable by mathematical operations. It is only through the application of kinship mathematics that those pending problems are solvable and the exploration of a new field of kinship studies can be attempted. The establishment of kinship science—KINOLOGY—is assured.

APPENDIX

THEORY OF GROUPS OF PERMUTATIONS, MATRICES AND KINSHIP: A CRITIQUE OF MATHEMATICAL APPROACHES TO PRES-CRIPTIVE MARRIAGE SYSTEMS

The applicability of mathematics to the study of Anthropology, especially to kinship structure, has long been discussed by social anthropologists. Studies to this aim have been undertaken especially in connection with the analysis of kinship terminology or section systems. These began toward the end of the last century so the term 'kinship algebra' has a long history.

Unfortunately, because most anthropologists are unfamiliar with mathematics, only pseudo-mathematics has been applied. Symbolic notations have been used, but these involve only description and no manipulation. Many papers have brought forth brilliant analyses and ingenious models, but a real scientific base for the mathematical approach has never been established. Not unexpectedly, the 'mathematical' results were extremely vague and scanty, and this has caused anthropologists to neglect the mathematical approach. The lack of a foundation in pure mathematics also made the leading scholars condemn kinship algebra as pseudo-science (e.g., Malinowski 1930). Thus for a long time kinship algebra has been treated as a step-child and subordinated to the traditional discipline, and no progress has been made in it. This stagnation continued until very recently.

Now a new breakthrough has come from an adjacent science. By introducing the method of 'componential analysis' from the field of linguistics the study of kinship algebra has been revived. The accomplishments of Lounsbury (1956, 1964), Hammel (1965) and others in the study of kinship terminology have restored its reputation. Also, through impulses from modern achievements of the natural sciences, anthropologists have been compelled to make epistemological and methodological reappraisals of their traditional methods. Then, through the advocacy of such outstanding anthropologists as Lévi-Strauss (1949, 1953) and Leach (1961), the new concept of 'structural models' or the problem of mathematical applications to kinship study again evoked serious attention. They have become primary topics in current anthropology.

One of the remarkable tendencies in modern mathematics is the developing emphasis on a qualitative orientation, in contradistinction to the quantitative point of view of traditional mathematics. As one outcome of this development, mathematics has directly or indirectly brought new methodological tools to the study of social sciences, which have been accepted widely and rapidly. Thus, in the field of anthropology, such subjects as mathematical logic, set theory, group theory, game theory, topology, cybernetics and others are thought indispensable for the study of kinship structure.

The realization of this has come mainly, not from anthropologists, but from mathematicians interested in kinship problems, especially in the section system of the Australian aborigines, exploring a new mathematical approach in this field.

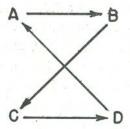
In 1949 André Weil, in the appendix to Part I of Les Structures

Elémentaires de la Parente by Lévi-Strauss, first applied real mathematics to analyze certain types of marriage laws. Proposing a new conception of 'marriage types', Weil points out that the theory of groups of permutations is applicable to the study of the section system. Since this time Weil's unique suggestion has shaped the mode of mathematical approaches to kinship study. But there are certain flaws in the approach, resulting from the misunderstanding of some anthropological phenomena. This may be partly caused by the fact that the mathematicians are not well informed of ethnographical resources. Hence Weil was trapped, through sampling error, into a discussion of the matrilateral cross-cousin marriage system based on a controversial example. This has not only influenced the theortical development of the field but also introduced the bias that has hindered the solution of unilateral cross-cousin marriage systems. mistakes of Weil and his successors which to date have not been clearly pointed out will be discussed in this paper.

Weil acknowledges that if a society practices the section ('class' in his usage) system the marriage laws have to satisfy the following conditions (White 1963: 151-157):

- (A) For each individual, male or female, there is one and only one type of marriage which he (or she) has the right to contract.
- (B) For each individual, the type of marriage which he (or she) is capable of contracting depends solely on his sex and the type of marriage of which he (or she) is the issue.

Then Weil takes a society of four sections practicing the following marriage rules as an example for his analytical method:



There are four types of marriage: (M_1) male A, female B; (M_2) male B, female C; (M_3) male C, female D; (M_4) male D, female A. Let us further allow that the children of a mother of class A, B, C, D be respectively of class B, C, D, A. Then our table is the following:

Type of parents' marriage M_1 M_2 M_3 M_4 Type of son's marriage $f(M_i)$ $=M_3$ M_4 M_1 M_2 Type of daughter's marriage $g(M_i)$ $=M_2$ M_3 M_4 M_1

f and g function to determine the type of marriage of a child issuing from a marriage of type M_i (i being one of the number 1, 2, ..., n). Moreover, in other words the second line and the third are simply arranged in a different order from that of the previous line. Thus it reveals that the theory of permutations is applicable to this study. Then Weil introduces a new condition:

(C) All males ought to be able to marry the daughter of their mother's brother.

For this, the following demonstration is given:

Let us express it algebraically. Consider a brother and sister, issues of a marriage of type M_i ; the brother must contract a marriage $f(M_i)$ so that his daughter will contract a marriage $g(M_i)$; the sister must contract a marriage $g(M_i)$ so that her son will contract a marriage $f(g(M_i))$; the condition (C) is thus expressed by this relation:

Weil's demonstration appears very reasonable, and seems beyond any argument. He postulates that a 'marriage cycle' which is necessary for the regulation of a unilateral cross-cousin marriage system has been established among the four sections, namely, by the rule that male A marries female B, male B marries female C, male C marries female D, and male D marries female A. Thus the marriage cycle penetrates the four sections and the above-mentioned demonstration proves that the marriage meets condition (C). That is, it is a matrilateral cross-cousin marriage, or to speak properly, a father's sister's son/mother's brother's daughter marriage. But, is Weil's assumption really valid? My answer is no. It is worth while to study Weil's scheme from a new angle. Let us examine the kind of descent rule that is practiced in the four section system drawn up by him.

In the male line, according to Weil's denotation, A's son is C, C's son is A, thus A and C are placed in the position of alternate generations or form a 2-generation cycle to compose one patri-descent line or group. In the same way B and D compose another parti-descent line or group. Weil's four sections now split into two even parts, each representing a patri-descent group. In the female line, where A's daughter is B, B's daughter is C, C's daughter is D, and D's daughter is A, the four sections are involved in a four generation cycle to compose one matri-descent line or group. Now we realize the society depicted by Weil is an extraordinary one; it is exogamous from the point of view of patrilineality, but endogamous from the point of view of matrilineality. This phenomenon is quite different from what is known for any 'four section' system, which is exogamous from

the point of view of both descent principles where sections of both descent groups intersect each other.

It is an established theory in social anthropology that at least three descent groups are necessary to form a marriage cycle for the practice of unilateral cross-cousin marriage. Weil's four sections take the form of a marriage cycle, but a section can not be substituted for a descent group, or vice versa. Weil's society is not composed of four but of two patri-descent groups; theoretically no 'remote exchange' can be set up between the two descent groups since no matrilateral cross-cousin marriage can take place. But in reality this is not the case. Let us reexamine Weil's marriage rule.

Consider a brother and a sister, issues of a marriage of type M_1 ; the brother must contract a marriage $f(M_1)=M_3$; the sister must contract a marriage $g(M_1)=M_2$ so that her daughter will contract a marriage $g[g(M_1)]=M_3$; thus a brother must contract a marriage with his sister's daughter. It is true for whatever type of marriage the parent belongs to. The above-mentioned condition is thus expressed by the following relation:

$$f(M_i) = g[g(M_i)].$$

In a word, the four section system discussed above is not a society organizing a marriage cycle by four descent groups and practicing matrilateral cross-cousin marriage as Weil thought, but a society composed of two patri-descent groups or one matridescent group and the underlying mechanism controlling the system is a kind of oblique marriage, i. e., 'brother/sister's daughter marriage' or 'uncle/niece marriage'.

If in a given society cross-cousin marriage between first cousins is prescribed, then it automatically follows that marriage

between second or more remote cousins is also found; similarly, if uncle/niece marriage is prescribed, then matrilateral crosscousin marriage should also occur in view of overlapping relationships resulting from the nature of the system itself. To characterize a system by a necessarily concomitant secondary feature, for instance to label the former as a 'second-cousin marriage system' or the latter as a 'matrilateral cross-cousin marriage system' leads to serious methodological error. Considerable confusion could result from thinking of a system such as that shown by Leach (1961:60, Fig. 6.) as one characterized by 'Patrilineal descent; marriage with m. B. d. and/or own sis. d.,' when it might better be thought of as 'uncle/niece marriage' or in the term of 'marriage with own sis. d. and/or m. B. d.' should be noted that Leach himself uses this diagram merely to illustrate overlapping, but my point is that it could be misleading. Similarly, Weil's explanation of matrilateral cross-cousin marriage using a system more basically thought of as characterized by uncle/niece marriage is also questionable.

In addition to this Weil proposes another case of a four section system to explain the definition of new terms, 'reducible' or 'irreducible' society, offered by him. If we follow out the marriage and descent rules given by Weil, we could recognize the society as being characterized by the possession of four matridescent or two parti-descent groups, and the system as regulated by an absurd 'father/daughter' or 'mother/son' marriage.

At the end of his article Weil introduces another mathematical device, the addition of an 'n-tuple modulo two system', to analyze the controversial Murngin system and support Lévi-Strauss's famous hypothesis. The Murngin system will be dealt with fully

in my article 'Formal analysis of prescriptive marriage system: the Murngin case', so this problem will be not discussed here.

Robert R. Bush, in his extremely significant extension of Weil's method (see White 1963, Appendix 2), concludes that the algebra of permutations, special topics in group theory, matrix algebra, and operator algebra are appropriate for the study of section systems. Thus Bush introduces the concept of a mathematical 'operator' demonstrating that 'permutation matrices' are an effective tool for analysis, and that the types of marriage X_1 , X_2 , ..., X_n form the vector $[X_1 \ X_2 \ ... \ X_n]$, and given the operators F and G (equivalent to Weil's function f and g) in the form of matrices. The operators are computable: If the product is FG=GF, this implies that matrilateral cross-cousin marriage is permissible. They are also computable with themselves to produce 'identity operator F, for example, $F^n=I$ of $G^n=I$. n represents the generation cycle for a given descent line; n is never larger than the number of elements in the vector operand.

In contrast to Weil, Bush makes use of actual societies such as Kariera, Tarau and Arunta, for which anthropological data exist. Among them the Tarau system is regulated by a matrilateral cross-cousin marriage system in which the number of sections and the number of descent groups coincide. In this case the marriage cycle consisting of its four sections is synonymous with the marriage cycle consisting of four patri-descent groups. This is the only possible type of matrilateral cross-cousin marriage system having four sections.

Developing Weil and Bush's theory and method, Kemeny, Snell and Thompson contribute some sections in their text-book, Introduction to Finite Mathematics (1956), to an algebraic analysis

of the Australian marriage rules. They first systematize the properties of the societies to be investigated as an integrated set of axioms. Then using these plausible axioms they show there are only a few allowed societies with a given number of marriage types. These are a major advance over the previous work, but there are still deficiencies in their discussion. First, they adopt an unsuitable example to discuss for the demonstration of matrilateral cross-cousin marriage. Second, the permutation matrices assigned for the operators S and D (equivalent to Bush's F and G) are given inversely. The latter mistake is carelessness, but the former one introduces a serious theoretical fallacy. In this case to adopt the equation, DS=SD, implying mother's brother's daughter/father's sister's son marriage, or matrilateral cross-cousin marriage, as the primary marriage rule is not proper. The lack of a criterion or axiom to distinguish the false item leads them to commit the same mistake as Weil.

The following properties are selected as valid:

$$S^3 = I$$
, $D^3 = I$, $S^{-1} = D$, $D^{-1} = S$.

They are to be read in anthropological parlance as follows:

"The three sections compose one patri-descent line and simultaneously compose a matri-descent line in reverse order to the former. Both descent line are regulated by the principle of the 3-generation cycle and characterized by the 'father/daughter' or 'mother/son' marriage."

Other equations such as $D^2=S$, or $S^2=D$ are secondary features, and they are valid as primary feature only under the condition that the given sections are divided into two to produce either patri- or matri-descent groups (cf. Weil's first case). Now the fallacy of the equation SD=DS as the primary feature becomes

evident, for no matrilateral cross-cousin marriage system can be established if only one, either patri- or matri-, descent group exists.

In contrast to the above-mentioned pioneer papers, an elaborate and important study of the prescriptive marriage system, An Anatomy of Kinship, with the subtitle of Mathematical Models for Structures of Cumulated Roles, was published by Harrison C. White in 1963. This book is comprised of three chapters. In Chapter 1 roles and kin trees are discussed, and the closed social structures of interlocked compound roles built by Australian aborigines are analyzed. Chapter 2 is the cream of this essay. All distinct kinship structures which satisfy the Kemeny-Snell-Thompson axioms are systematically derived and described, and a simple type of graph is used to depict the results for cases typical of each subtype for each of the four major classes (bilateral, matrilateral, patrilateral and others) of kinship systems. Chaptor 3 the major field report on each of several well-known tribes is analyzed to see what ideal model of a closed and consistent kinship structure best fits existing data. At the end of the book Weil's article (now translated from French into English) and Bush's unpublished mimeo are reprinted as appendices.

Perceiving that marriage type is not a concept to be found in the field notes of anthropologists or the thinking of members of the societies, White suggests that a considerable reformulation of the Kemeny-Snell-Thompson approach is desirable. Instead of having one matrix represent the transformation of parent's marriage type into son's type, and another similar matrix represent daughter's marriage type, White deals with one matrix for transforming husband's section (clan in White's usage) into wife's

section (W), and another for transforming father's section into children's section (C). Using both W and C as generators, the kinship structure of all derivations is interpreted mathematically. Can White's reformulation eliminate the fallacy introduced by his predecessors? Let our examination concentrate on the item of Chapter 2 Section 8 (White 1963: 52-56), where the matrilateral marriage system is discussed:

Assume that WC=CW, but $W^2\neq I$ so that $WC\neq CW^{-1}$ and $CW\neq W^{-1}C$..., let q be the order of W and p the order of C. Thus $C^p=I$, and $W^q=I$, q>2.

This is the condition proposed by White for the matrilateral marriage system. It is necessary, but not sufficient. Then three types of matrilateral marriage system are derived from this theorem:

- (1) Assume $W=C^j$, where j is the smallest positive integer for which the equation holds. Then $1 \le j < p$; p > 1; and also $j \ne p/2$, or else bilateral cross-cousin marriage would be allowed.
- (2) Assume $W \neq C^j$ but $W^2 = C^m$, where $1 \leq m < p$, since m = p corresponds to bilateral marriage, and p > 1 for the same reason.
- (3) Suppose $W^a \neq C^b$ for any $1 \leq a < c$, no matter what b is, $1 \leq b < p$. Then assume $W^c = C^m$ for $1 \leq m \leq p$. (Only in Case 2, where c = 2, is m = p excluded, so as to rule out bilateral marriage.)

Among them only Case 3 is valid, and the other two are questionable. Let us examine one of the diagrams shown in White's Fig. 2.13.a (1963: 53) for j=2 and p=5 for Case 1. According to the designations of the diagram, the following permutation matrices for C and W may be derived:

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad W = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

By the multiplication of the operators or generators, we could get the following equations:

(a)
$$C^5 = I$$
 (b) $W^5 = I$ (c) $W = C^2$ (d) $W^2 = C^4$

(e)
$$W^3 = C$$
 (f) $W^4 = C^3$ (g) $WC = CW$.

These equations imply: The five sections compose one patridescent group (a) and practice grand-father/grand-daughter marriage (c); the other five equations are concomitant secondary features which are valid as the primary feature only under the condition that the numbers of the power of W are equal to the numbers of patri-descent groups (b, d, e, f) or more than three patri-descent groups exist (g) in the given society. Under the condition of (a) these five are all unfeasible as primary ones.

In the same way we could demonstrate that the society given in White's Fig. 2.13.b is regulated by grand-mother/grand-son marriage.

For Case 2, the four section society depicted in Fig. 2.14 (1963: 54), is regulating by following permutation matrices for C and W:

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad W = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Through the manipulation of the operators, the following equations are gained:

(a)
$$C^2 = I$$
 (b) $W^4 = I$ (c) $W^2 = C$ (d) $WC = CW$

These equations imply: The four sections are divided into two parts and compose patri-descent groups respectively; they are regulated by the principles of alternating generations or 2-gene-

ration cycles (a). An oblique marriage, namely uncle/niece marriage, is practiced by them (c). The other two equations (b) and (d) are invalid for this case. The former needs four parti-descent groups and the latter three or more parti-descent groups.

In the same way the societies depicted in White's Fig. 2.15a and b (1963: 55), could be both demonstrated to be composed of two patri-descent groups and regulated by uncle/niece marriage.

From the discussions above, we have seen that the mathematical approach proposed by Weil and developed by the mathematicians does not allow distinction between oblique marriages—in this case they are micro or incestuous ones—and cross-cousin marriages. This defect is caused by the confusion of sections and descent groups. The Kemeny-Snell-Thompson axioms or those revised by White are apparently insufficient. Thus modification and clarification are desirable. Whether the new principle of the 'marriage cycle' proposed by Russell M. Reid (1967) as the ninth axiom could cover the deficiency is worth examining.

Reid admits that the marriage cycle is the essential feature of the model resulting from White's eight axioms and defines it as "closed and nonintersecting loops of wife-taking ties connecting a fixed finite number of segments of a marriage system." Thus the ninth axiom postulates that all such cycles in the same system must contain the same number of segments. This axiom is necessary to satisfy the requirement that the algebraic group contains a unique element I. In the situation just described, $I=W^m$, where m is the number of segments in each cycle. Reid translates this equation into nonalgebraic terms as that m successive wife-taking links will always complete a loop. Then

Reid concludes that for all of the Australian examples m is 2. This conclusion is not always valid, for it denies the existence of unilateral cross-cousin marriage systems. White has already pointed out that $I \neq W^2$ is the basic condition for the realization of unilateral cross-cousin marriage systems (1963: 42). The equation implies that at least three descent groups are necessary for an organization of the marriage cycle. But this property seems to be ignored or overlooked by the proposer. Reid's new axiom has the latent faculty to clarify it, but owing to his ignoring of the unilateral cross-cousin marriage system and the relationship between marriage rules and descent groups, Reid missed the opportunity to realize it.

Another proposal of Reid of adopting new generators F and M instead of White's C and W is constructive, as F defines the relative positions of individuals with their fathers and M likewise with their mothers. But mathematically these new generators produce quite the same effects as those proposed by Bush and his successors. In this case the abolishment of the generator W causes Reid's ninth axiom to become useless.

In spite of this criticism it is not my intention to condemn the mathematical approach and deny its merits. Once its faults are clarified—to which aim an attempt has been made above—the mathematical approach proves to be highly efficient for the advanced study of kinship theory. As compared with the astonishing quantity of field data accumulated, kinship study is still stagnating in the descriptive stage, and its theoretical accomplishment seems poor. For example, for such basic problems as the origin of incest taboo, the formation of genealogical space and its structure, the genetical study of the section system or

other descent groups, no acceptable hypotheses have ever been proposed or even discussed. The mathematical method has only taken its first step and its effective range is very limited; however, considering the role played by mathematics in the field of natural sciences, the results to be expected by its application for anthropology surpass our imagination. If it is our ultimate aim to establish kinship study as a pure science, there is no reason why anthropologists should hesitate to accept mathematical method. It is true as Leach (1964) has already pointed out that most anthropologists are quite unfamiliar with mathematics. But the anthropologist should attempt to conquer the mathematical difficulties and at the same time it is hoped that mathematicians on their part concern themselves with anthropolgical problems. Only a cooperation between anthropologist and mathematician can successfully explore the limitless scope of kinship study.

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孟根的親屬結構

一個數學方法的研究

劉斌雄

中華民國五十九年九月 臺北 南港

Errata

PAGE	LINE	WRONG	CORRECT
5	2	began	begun
18	17	socity	society
18	25	discuse,	discuss,
51	24	conbining	combining
51	27	deduced	reduced
60	23	4-gneration	4-generation
66	16	$S_2=f^0m$,	$S_2=f^3m$,
70	6	participting is	participating in
82	(Table 11)	kamiyer	kaminyer
84	21	becom	become
85	27	X_5	X_4
95	21	f^{2n+2}	f^{2n+3}
95	24	depicated	depicted
95	28	sytem	system
104	11	marriage G4	marriage alliance G4
106	24	conbinations	combinations
107	8	X_1, Y_1, X_3, Y_4 and X_5	X_1 , Y_1 , Y_2 , X_3 , Y_4 and X_5
110	23	separately.	separately,
122	4-5	daughther's	daughter's
122	9	feiled	failed
130	15	of	or

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